Preliminary draft
November 10, 2010

Marking-to-Market Government Guarantees to Financial Systems An Empirical Analysis of Europe

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Abstract. *We propose a new index for measuring the systemic risk of default of the banking sector, which is based on a homogeneous version of multivariate intensity based models (Cuadras – Augé distribution). We compute the index for 10 European countries, exploiting the information incorporated in the CDS premia of 44 large banks over the period January 2007 – September 2010. In this way, we provide a market based measure of the liability incurred by the Governments, due to the implicit bail-out guarantees they provide to the financial sector. We find that during the financial crisis the systemic component of the default risk in the banking sector has significantly increased in all countries, with the exception of Germany and the Netherlands. As a consequence, the Governments' liability implicit in the bail out guarantee amounts to a quite relevant share of GDP in several countries: it is huge for Ireland, lower but still important for the other PIIGS (Italy is the least affected within this group) and for the UK. Finally, our estimate is very close to the overall amount of money already committed in the rescue plans adopted in Europe between October 2008 and March 2010, despite strong cross-country differences: in particular, Germany and Ireland seem to have committed an amount of resources much larger than needed; to the contrary, the Italian Government has committed much less than it should.*

1. Introduction

In the current phase of the financial crisis, the question attracting most of attention and debate is the mutation and transmission of the credit virus from the banking system, where the epidemic started, to the Governments' balance sheets. According to some interpretations of the crisis, the diffusion of the virus across financial institutions and markets was magnified by the accounting rules that were imposing to *mark-to-market* assets and liabilities of all the players in the financial system. Someone could argue, with somewhat of a joke, that the most acute phase of the crisis was over when the problem exposures ended up in the only budget that is not evaluated at fair-value yet, namely the Government budget. This argument calls for a question: if the implicit guarantee provided by each Government to its financial system were marked-to-market, what would be the impact on public debt figures? This paper addresses this question, focusing on the situation of Europe. Addressing this question amounts to asking how much each Government should pay in order to buy insurance against default of one financial institution or even of the whole financial system. This can be done, as it is done for budgets of companies, by extracting this cost from market quotes, in this case the values of *Credit Default Swaps* (CDS) written on "names" of financial institutions. Notice that this question has nothing to do with arguments concerning whether the CDS markets provide faithful representations of credit risk, or whether their forecasting power is hampered by the risk premium embedded in them. No matter if one finds this risk premium excessive or not, this is the market price a Government should pay to buy protection against default of the financial system.
This issue is relevant not only for academics and financial practitioners. It is becoming increasingly important in the policy debate on the reform of the Stability Pact in Europe, where some policy-makers argue that private debt should be included in the evaluation of the financial soundness of

member countries. Our paper contributes to this debate by introducing a methodology to estimate – using market data – the burden to be added to the debt/GDP ratio in each European country, due to the implicit bail-out guarantee that Governments provide to the financial sector. Following the dramatic consequences of the Lehman Brothers default, many Governments have announced and implemented rescue plans for the financial sector, creating the expectation of a wide bail-out policy. This expectation clearly emerges from CDS quotes: the default risk priced by the market shows a remarkable co-movement between the financial sector and the Government sector.

The plan of the paper is the following. After reviewing the literature most closely related to our work (section 2), we lay out a multivariate intensity based model of credit risk (section 3), which provides the framework for our empirical methodology to measure the fragility of the financial sector of a country (section 4): we introduce here a new financial stability index (named "Cuadras-Augé index"). In section 5 we present our empirical evidence, using a sample of 44 large banks located in 10 European countries: among other things, we provide here an estimate of the liability incurred by the European Governments, due to the implicit bail-out guarantee. Section 6 summarizes our main results. The detailed derivation of the financial stability index is postponed to Appendix 1. Finally, Appendix 2 provides detailed information on our sample and further analysis at country level.

2. Related Literature

The idea that the implicit guarantee of bail out, provided by the Government to the financial sector, should be taken into account in the balance sheet of the public sector has been developed by Gray, Merton and Bodie (2006), and by Gapen, Gray, Lim and Xiao (2005). They apply the contingent claim analysis, where the bail out guarantee – particularly relevant for the "too-big-to-fail" intermediaries – is modelled as a put option enabling a bank to sell its own assets to the Government, which pays a strike price equal to the value of the bank liabilities backed by the guarantee. They provide a methodology for quantifying the value of the put option, which should be included in the asset side of the balance sheet of financial intermediaries and in the liability side of the public sector balance sheet. Although different from our work on technical grounds – they take a structural approach while we use an intensity based model – they share with us the idea that an accurate measurement of sovereign risk cannot ignore the interlinks between the different sectors of the economy, in particular between the financial and the public sectors.

The recent financial crisis has dramatically increased the need to asses the impact of the bail out measures taken by several Governments in order to avoid the collapse of the financial system. Through these measures – capital injections, debt guarantees, asset purchases and asset guarantees – the public sector has committed a huge amount of resources in several countries (we report some data below in section 5.3). The overall impact of this kind of interventions has been positive as far as the credit risk of financial institutions is concerned. BIS (2009) shows that the announcement and the implementation of rescue packages have been followed by a fall in bank CDS spreads, so they have indeed been able to reduce the default probability of banks as perceived by the market. However, they have at the same time increased the market price of sovereign risk. Ejsing and Lemke (2009) show that, following the announcement of rescue packages in the fall of 2008, a marked increase of sovereign CDS spreads has come along with the reduction of bank CDS spreads.

The analysis by Berndt and Obreja (2010) is closely related to ours. They study the correlation structure of weekly CDS returns for a large sample of European firms over the period 2003 – 2008, showing that the co-movement among them increases dramatically after the onset of the financial crisis in August 2007. They trace back this result to the larger weight of a factor mimicking the economic catastrophe risk. These findings are in line with ours, which point to a significant increase of the systemic component in the default risk of European banks during the financial crisis, as we shall see below.

Finally, our work contributes to the debate on the sustainability of public debt. Starting with the seminal article by Hamilton and Flavin (1986), a number of studies have analysed the ability of the public sector to keep its debt on a sustainable path over time, namely a path consistent with the inter-temporal budget constraint[1]. The approach taken here suggests that the rescue plans and the implicit guarantee of future bail outs in support of the financial sector create a liability for the public sector, which should be taken into account in assessing the sustainability of the public debt.

3. The multivariate intensity based model

It is well known that credit risk models can be built either on the basis of economic information on the business activity and the balance sheet of the obligors, or on statistical information based on the probability distribution of the default event (*default probability*, DP) and the loss incurred in case of default (*loss given default*, LGD). The former class of models is referred to as the *structural approach* to credit risk, while the second is known as the class of *reduced form* or *intensity based models*. In cases in which the default of the obligor may occur for reasons that do not depend, or not depend only, on the balance sheet structure, the reduced form approach is clearly preferable. This is actually the case when financial institutions are the obligors. As recent experience proves (and the ancient confirms), the default of a financial institution may occur for many reasons that are not strictly dependent on the balance sheet, such as a panic or a bank-run, or contagion and liquidity crises. For this reason, the model that we use to represent the credit risk of financial institutions, and of a financial system, is the simplest multivariate extension of an intensity based approach: this is called the Marshall-Olkin (1967) model. This model was first applied to credit risk by Esposito (2002), and was recently extended to a hierarchical structure by Durante et al. (2009).

The idea is that the default probability of each financial institution is modelled as a Poisson process with intensity equal to $\hat{\lambda}_i$. We remind that intensity is the instantaneous percentage increase of probability of an event, in our case the default event:

$$dP_i(t) = \hat{\lambda}_i P_i(t) \tag{1}$$

where $P_i(t)$ is the probability of *default*. In the most straightforward and easy application the intensity may be considered constant or conditionally constant, that is constant given the information available. The probability of no event occurring by a given time T, that in our case is the probability of survival of the obligor, is then given by the simple formula

$$P_i(\theta_i > T) = \exp\left(-\hat{\lambda}_i (T - t)\right) \tag{2}$$

where θ denotes the default time. The default probability is of course the complement to one. The natural multivariate extension of this approach is to consider the probability of an event that causes the default of more than one obligor. So, the default intensity of each obligor should be decomposed into

$$\hat{\lambda}_i = \lambda_i + \xi_i \tag{3}$$

where λ_i denotes the idiosyncratic components and ξ_i collects the intensities of all events triggering default of the obligor alongside of others. Specifying ξ_i is actually the major problem with this model, since it would call for the default of all possible combinations of obligors. Typical shortcuts

[1] See Baglioni and Cherubini (1993) for further references on this topic and for an application to Italy.

are to limit the dependence structure to the bivariate setting or to consider only a single factor triggering default of all the obligors. The latter solution seems the fittest one for our problem, since it would represent the risk of a systemic crisis of the banking system. In this case, we would set $\xi_i = \overline{\lambda}$ for all the financial institutions in the sample. Notice that imposing a specification amounts to select a specific dependence structure to the model. The correlation among default times of any pair of obligors is in fact given by

$$\rho_{ij} = \frac{\lambda_{ij}}{\lambda_i + \lambda_j + \lambda_{ij}} \tag{4}$$

where λ_{ij} denotes the intensity of default of both obligors. In the systemic risk specification suggested above, this would boil down to

$$\rho_{ij} = \frac{\overline{\lambda}}{\lambda_i + \lambda_j + \overline{\lambda}} \tag{5}$$

One could also specify the dependence structure separately from the probability of default of each obligor by extracting from the Marshall-Olkin distribution the corresponding copula function (see Nelsen, 2006 for a broad review). We remind that copula functions allow to specify separately the marginal distributions of variables and their dependence structure. The Marshall-Olkin copula turns out to be

$$\hat{C}(u_1, u_2, ..., u_n) = u_1 u_2 ... u_n \min\left(u_1^{-\alpha_1}, u_2^{-\alpha_2}, ..., u_n^{-\alpha_n}\right) \tag{6}$$

where u_i are uniformly distributed variables and α_i are dependence parameters given by

$$\alpha_i = \frac{\overline{\lambda}}{\hat{\lambda}_i} \tag{7}$$

The joint survival probability of the credit exposures in a basket (that is the probability that default would not occur in a given period) can then be represented by computing $\hat{C}(1 - DP_1, 1 - DP_2, ..., 1 - DP_n)$, where DP_i is the default probability of the *i*-th obligor. Notice that the parameter α_i has an important interpretation in terms of systemic relevance of each obligor, meaning that the obligor for which this parameter is very close to 1 carries very little idiosyncratic risk. Notice that the notion of different α_i values for different obligors correspond to the concept known in statistics as non-exchangeability, meaning that $\hat{C}(1 - DP_i, 1 - DP_j) \neq \hat{C}(1 - DP_j, 1 - DP_i)$. If one wished to endow the Marshall-Olkin copula with the exchangeability property, he would have to set $\alpha_i = \overline{\alpha}$ for all exposures. This is known as *Cuadras-Augé copula* or *exchangeable Marshall-Olkin*. As it is well known, copulas are non parametric representations of association among variables, and as such they are naturally linked to non-parametric measures of association. The measures that are used the most are Kendall's τ concordance index and Spearman's ρ rank correlation measure. Copula function parameters can be actually calibrated on these measures. In the case at hand, Kendall's τ can be computed to be

$$\tau_{ij} = \frac{\alpha_i \alpha_j}{\alpha_i - \alpha_i \alpha_j + \alpha_j} = \rho_{ij} \tag{8}$$

and, only for the Marshall-Olkin case, is equal to the correlation figure. As for Spearman's ρ, one can compute instead

$$\rho_{s,ij} = \frac{3\alpha_i\alpha_j}{2\alpha_i - \alpha_i\alpha_j + 2\alpha_j} \tag{9}$$

In the case of *Cuadras-Augé* copula $\alpha_i = \overline{\alpha}$ implies $\tau_{ij} = \overline{\tau}$ and $\rho_{s,ij} = \overline{\rho}_s$ for all pairs. So, the Cuadras-Augé copula is well suited to represent the joint default probability distribution of a homogeneous set of exposures, where by homogeneous it is meant that the exposures have all the same probability of default and the same bivariate dependence structure.

4. The Cuadras-Augé Financial Stability Index

Having laid out the basics of the multivariate intensity based model, we propose here the methodology of our empirical analysis. Our task is to extract a synthetic financial stability index representing the creditworthiness of the financial sector of a country. The index must reflect both the average level of risk of each financial institution in the system and the correlation among them. Possibly, the index should be able to represent the degree of systemic and idiosyncratic risk in the financial system of a given country. As a warning, remember that this task is entirely different from that of specifying and estimating the joint probability distribution of the system. When we build a financial stability index what we are actually doing is to adapt a model, typically a homogeneous credit exposure model, to provide a close enough, albeit synthetic representation of risk. As the most famous example, one of the first indexes that was proposed in the industry, from the Moody's rating agency, was called the *diversity score*. This index provided a representation of the credit risk of a portfolio in terms of a basket of homogeneous independent exposures. In this case, homogeneous only means that each exposure has the same probability of default, that is actually meant to represent the average credit risk in the basket. Other financial stability indexes are computed and published by the IMF on a regular basis.

We propose a new financial stability index, which is based on the homogeneous version of the multivariate intensity based models, that as we saw is called Cuadras-Augé distribution. For this reason we call it "Cuadras-Augé index". The index is synthetically represented in terms of a marginal intensity $\hat{\lambda}$ which is the same for all the exposures in the set, and a dependence structure that is the same across all pairs of exposures (measured either in terms of concordance index, rank correlation or the $\overline{\alpha}$ parameter). A natural choice to calibrate the index is to set:

i) the marginal intensity equal to the average marginal intensity of the basket, and
ii) the correlation figure equal to some average of the figures in the correlation matrix.

Collapsing the correlation matrix in a single figure is common usage in the basket credit derivatives market, where for some products the implied correlation is also used as a quoting device. The main difference with respect to our model is that while the gaussian copula is typically used in the market, here we apply the Cuadras-Augé one. It is also worth noting that there is an interesting symmetry in the fact that while the gaussian copula correlation stems directly from structural models of credit risk, the Cuadras-Augé is the direct offspring of intensity based models. Here we report the main idea. We start from the Pearson correlation formula

$$\rho_{ij} = \frac{\lambda_{ij}}{\lambda_i + \lambda_j + \lambda_{ij}} \tag{10}$$

for each pair and we want to come up with a representation of the average dependence of the system based on

$$\rho = \frac{\overline{\lambda}}{2\hat{\lambda} + \overline{\lambda}} \tag{11}$$

for all the pairs. The straightforward choice for marginal intensities is to set them equal to the average, namely

$$\hat{\lambda} = \hat{\lambda}_m = \frac{\sum_{i=1}^{n} \hat{\lambda}_i}{n} = \frac{\sum_{i=1}^{n} \lambda_i}{n} + \overline{\lambda} \tag{12}$$

As for the solution of this problem, in Appendix 1 it is shown that if the harmonic mean is used to represent average correlation, this yields a filter that amounts to the figure $\overline{\alpha} \equiv \overline{\lambda} / \hat{\lambda}_m$ in the Cuadras-Augé system, where

$$\overline{\lambda} = \frac{2}{1 + \frac{1}{H}} \hat{\lambda}_m \tag{13}$$

and H denotes the harmonic mean of the Pearson coefficients. The same procedure could be applied to rank correlation, remembering that in Marshall-Olkin structures we have

$$\rho_{s,ij} = \frac{3\lambda_{ij}}{2\left(\hat{\lambda}_i + \hat{\lambda}_j\right) - \lambda_{ij}} \tag{14}$$

where $\rho_{s,ij}$ denotes rank correlation. Using the same procedure explained in Appendix 1, one obtains

$$\overline{\lambda} = \frac{4}{3} \frac{1}{\frac{1}{3} + \frac{1}{H_s}} \hat{\lambda}_m \tag{15}$$

where H_s denotes the harmonic mean of the rank-correlation coefficients.

5. Empirical analysis

5.1 The data set

We report here a general description of the data set, which is described in greater detail in Appendix 2 (Table A1). We collected data for the banking systems of 10 European countries, with a total of 44 banks. The selected banks were those used for the stress test exercise performed by the Committee of European Banking Supervisors (CEBS) in July 2010. From this sample, we dropped those banks for which Credit Default Swap data were not available. Our sample includes the banks with the highest systemic relevance in Europe, since all the major institutions are included, accounting for a large market share in each country. For all the banks in the sample, as well as for Governments, we collected a data set of CDS spreads for the 5 year maturity; the sample period goes from January 2007 to September 2010.

The marginal intensities were computed using the standard approximation rule

$$\hat{\lambda}_{it} = \frac{CDS_{it}}{LGD} \tag{16}$$

where CDS_{it} denotes the *CDS* quote of the *i*-th name at time *t*. By market convention, the reference *LGD* figure used to compute the price is set equal to 60%. Once the five-year intensity figure has been extracted, we computed survival probabilities of all the names in the sample. Then, for each country we reckoned the rank correlation figure using a rolling window of one year of data. This way, for most of the countries we have a time series of rank correlation matrices starting from January 2008. In cases in which data quality was particularly poor, the time series started later. In all cases, we have all the data for 2009 and 2010 (with the slight exception of Greece for which the first quarter is missing). Having done this, we applied the Cuadras-Augé filter described in section 4 to represent an index of the systemic risk component. Concerning the computation of the filter, we faced the problem of some negative correlation figures, while the computation of harmonic mean can only be computed for positive variables. In this case, we could have either excluded these negative figures from the computation of the mean or computed the arithmetic mean instead of the harmonic one. We decided to follow the latter route. This leads to an overvaluation of the filter, because the harmonic mean is lower than the arithmetic one: however, this effect is somewhat mitigated by the fact that negative correlations bring about a decrease in the value of the mean.
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5. 2. The Probability of a Banking Crisis

In figure 1 we report the intensities of a systemic event for our sample. It is easy to note an upsurge of this risk in February 2009 when the effects of the banking crisis of 2008 displayed their effects in full. Following the Greek crisis the increase in the probability of a banking crisis reached heights that had never been seen before, and not only for Greece. The countries that are exposed the most are, beyond Greece, Ireland, Portugal and Spain. The other countries are clearly part of a different cluster, with Italy, Austria and UK with the highest risk among those countries.

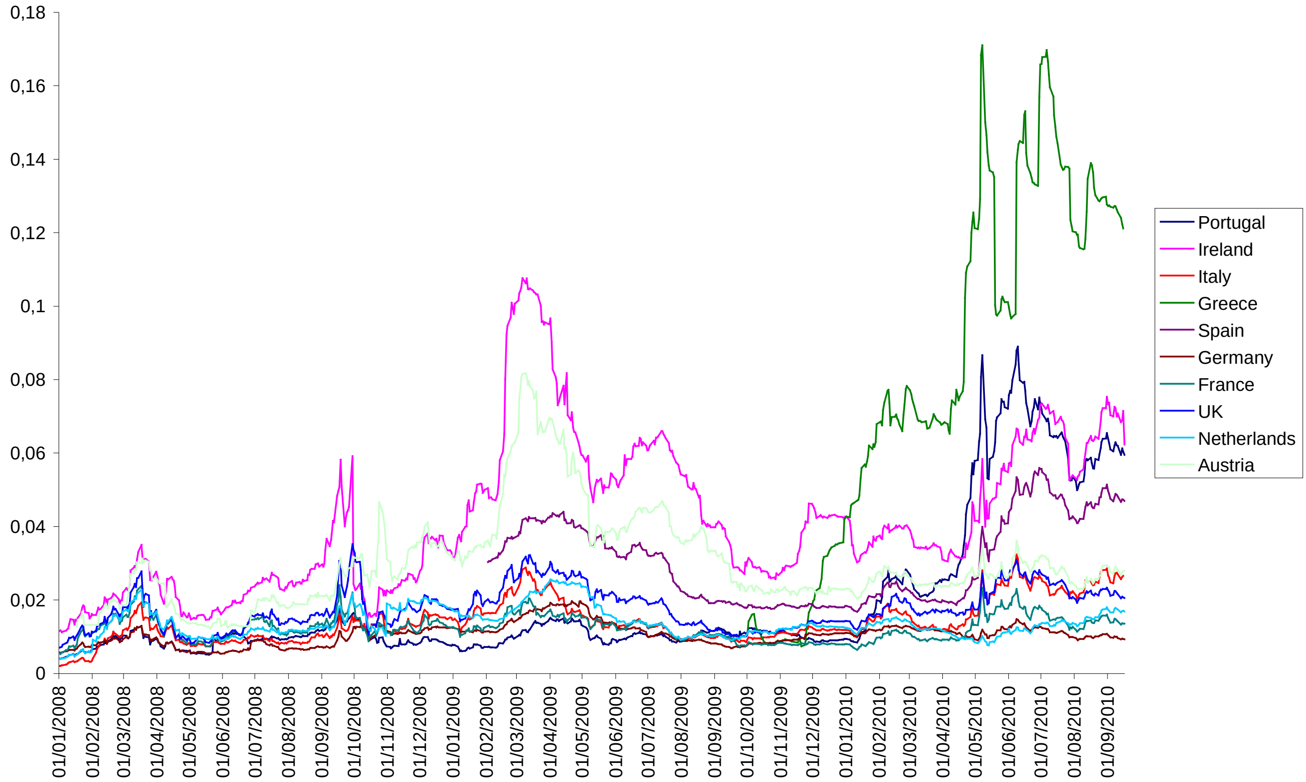


Figure 1. Systemic intensities. Financial sectors of selected countries

	Table 1. Risk adjusted probability of a systemic shock to the banking system					
Date	**17/03/2008**	**15/09/2008**	**16/03/2009**	**15/09/2009**	**15/03/2010**	**15/09/2010**
portugal	6,18%	7,05%	6,47%	5,26%	10,15%	26,06%
Ireland	16,06%	21,46%	40,61%	15,97%	15,70%	30,05%
Italy	9,22%	6,03%	11,86%	4,43%	6,17%	12,42%
Greece					29,02%	45,45%
Spain			19,03%	9,20%	9,49%	21,06%
Germany	6,27%	4,43%	8,12%	3,46%	5,85%	4,57%
France	11,27%	8,97%	8,82%	4,83%	4,56%	6,56%
UK	12,96%	10,07%	13,91%	5,10%	8,05%	9,85%
Netherland	10,79%	7,65%	10,45%	4,46%	5,84%	8,15%
Austria	14,01%	12,14%	31,96%	12,58%	11,42%	13,02%

Table 1 reports the values of the probability of a systemic shock priced in the CDS for each country. Figure 2 enables to appreciate the relevance of the systemic risk out of the overall average risk. The dynamics of the filter represents the ratio of the systemic to the average intensity, which is actually the $\overline{\alpha}$ parameter of the Cuadras-Augé formulas. This can be actually seen as a dependence parameter of the financial sector of each country. The main feature emerging from the graph is the recent increase of the systemic content of default intensity for all countries, with the noticeable exceptions of Germany and the Netherlands (see also Table 2 below). The counterparty of this evidence is that for all the countries, where we have found an increase of average rank correlation in the latest period of the sample, we also observe an increase of concentration of the correlation values between pairs (see Appendix 2 for information on individual countries).

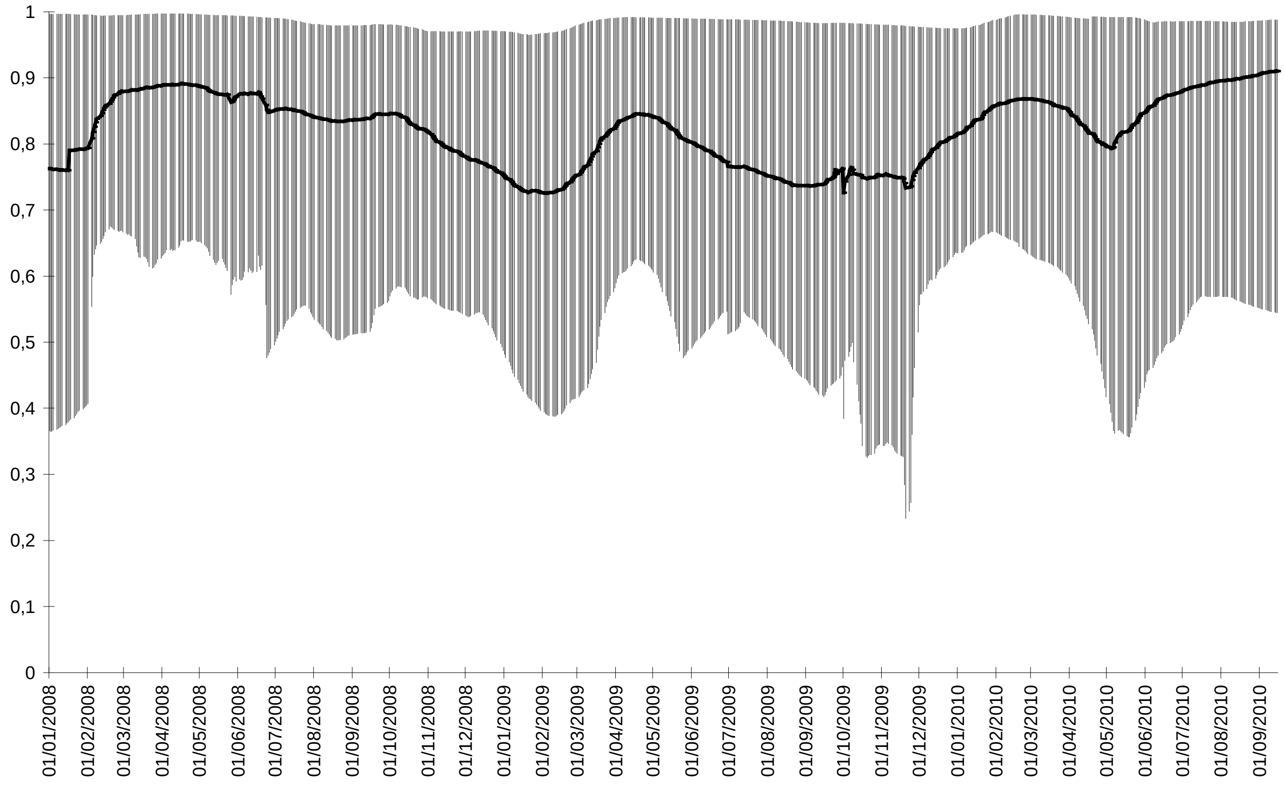


Figure 2. Cuadras-Augé filter. Maximum, minimum and average values. 10 European countries

Table 2 reports, for the same dates as those in Table 1, the values of the filter $\overline{\alpha}$. The economic meaning is the percentage relevance of a systemic shock out of the average probability of a credit event. Note that in the final observations of the sample the filter is very close to 1 for all countries, except Germany and the Netherlands. The case of Germany is peculiar, since the filter is around 55%, much lower than anywhere else.

	Table 2. Relevance of the systemic event					
Date	**17/03/2008**	**15/09/2008**	**16/03/2009**	**15/09/2009**	**15/03/2010**	**15/09/2010**
Portugal	64,14%	63,81%	48,68%	69,07%	89,69%	98,62%
Ireland	99,65%	97,71%	98,99%	98,39%	99,48%	98,91%
Italy	90,19%	82,16%	87,61%	85,75%	93,59%	96,14%
Greece					97,06%	99,11%
Spain			83,96%	67,32%	63,97%	95,78%
Germany	66,03%	52,69%	66,71%	42,77%	69,59%	55,51%
France	99,10%	98,05%	89,21%	84,59%	93,19%	98,24%
UK	99,05%	94,29%	90,82%	71,61%	93,28%	94,95%
Netherland	92,77%	88,41%	57,50%	55,06%	69,54%	80,03%
Austria	99,78%	98,19%	97,28%	97,15%	99,64%	96,91%

5. 3. The Bail-out Cost of a Banking Crisis

European Governments have committed a huge amount of money to restore public confidence in the financial sector, starting from the most acute phase of the financial crisis, namely in the aftermath of the Lehman Brothers collapse. In the period running from October 2008 through March 2010, the member States of EU-27 have committed an overall amount of resources equal to 4,131 bn euro, equivalent to 32.6% of their GDP[2]. Table 3 reports a breakdown of the intervention measures taken by the Governments of the ten countries covered by our analysis.

The bulk of the involved resources have been devoted to *guarantee schemes*. Governments have relied extensively on this tool, particularly until mid-2009, as it is the most cost-effective way for restoring the confidence of investors. Banks' liabilities are backed by the guarantee provided by the State; at the same time the Government budget is not hit by an immediate outlay. The "take-up rate" – the actual use of funds relative to the allocated amounts – is on average 32%, but it is much higher in some individual countries (like Portugal: 51%).

Many banks turned out to be under-capitalized during the financial crisis. Governments have reacted by approving both *recapitalization schemes* for the bank sector as a whole and *ad hoc* measures for individual troubled institutions. The take-up rate differs remarkably between the two kinds of intervention: 27% for schemes and 90% for *ad hoc* cases. The reason is that individual measures have been designed to meet well defined and urgent needs to restore a sufficient capital base of some institutions.

Two countries, namely UK and Ireland, account for the bulk (80%) of the third kind of intervention: *impaired assets relief*, where the Government either provides an insurance against assets devaluation or it directly buys some bank troubled assets. While Ireland has approved a major asset relief scheme, the intervention in UK has been on an individual basis.

[2] See EU (2010).

Table 3 - Government commitments supporting the financial sector (Oct. 2008 – March 2010)
(billions euro)

	Guarantee schemes	Recapitalization schemes	Other (*)	Individual cases	TOTAL
AUSTRIA	75	15		0,5	**90,5**
FRANCE	265	23,95		62,2	**351,15**
GERMANY	400	80		107,6	**587,6**
GREECE	15	5	8		**28**
IRELAND	376		54	25,6	**455,6**
ITALY		20			**20**
NETHERLANDS	200			56,2	**256,2**
PORTUGAL	16	4		0,5	**20,5**
SPAIN	200	99	30		**329**
UK	381,87	62,79		405,6	**850,26**
TOTAL	**1928,87**	**309,74**	**92**	**658,2**	**2988,81**

Source: EU (2010)
(*) Liquidity and asset relief schemes

Table 4 shows – for each of the countries included in our sample – the overall cost of the different kinds of intervention outlined above, both as a proportion of the banking sector size and as a ratio to GDP. The numbers reported in the table highlight the existence of great differences across countries. Ireland has committed an incredibly large amount of resources, both in terms of bank assets and as a ratio to GDP. It is followed by UK and the Netherlands: they have implemented interventions for amounts equivalent to about a half of their GDP. All the other countries have committed important amounts of resources, putting at stake a significant share of their GDP, although lower than in such extreme cases. The only exception is Italy: the Italian Government distinguishes for having spent a negligible amount of money.

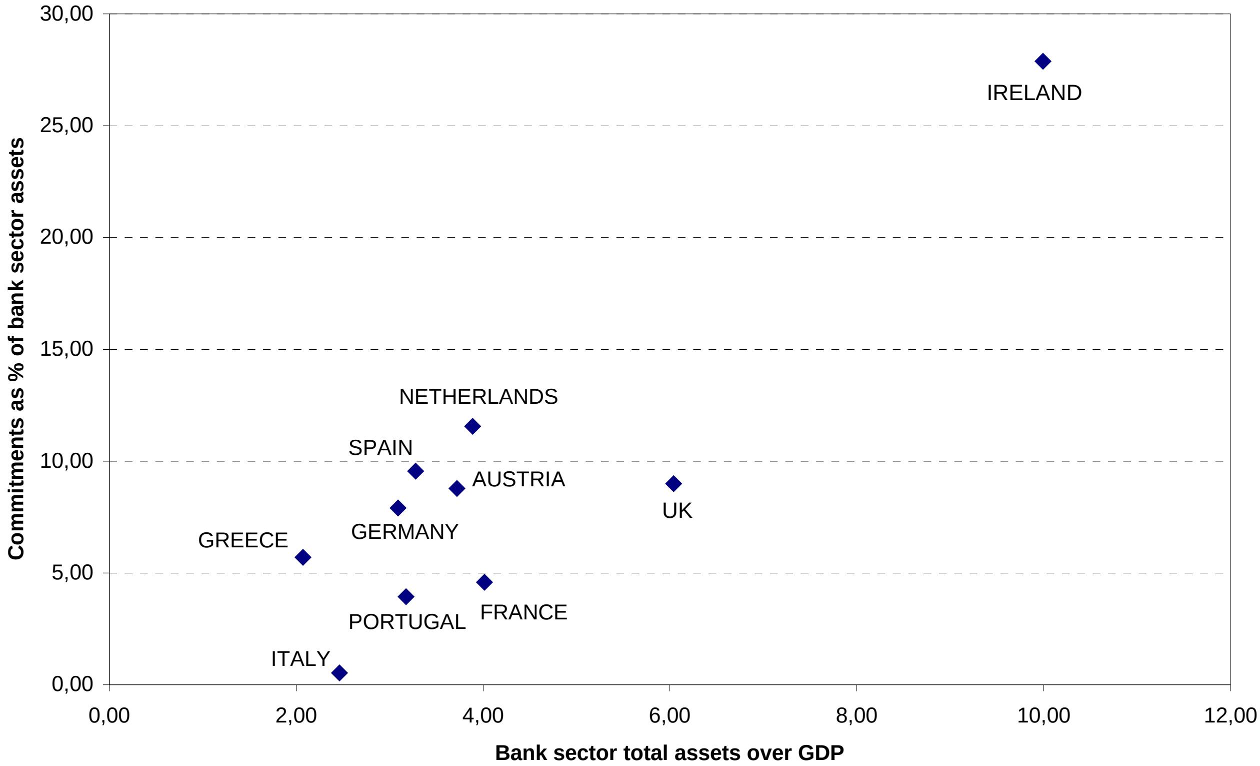
Table 4 – Government commitments over bank assets and GDP

	Government Commitments (% of Bank Sector Total Assets) (A)	Bank Sector Total Assets over GDP (B)	Government Commitments (% of GDP) (A*B)
AUSTRIA	8,78	3,72	32,68
FRANCE	4,59	4,01	18,41
GERMANY	7,90	3,09	24,41
GREECE	5,69	2,07	11,79
IRELAND	27,88	9,99	278,58
ITALY	0,53	2,46	1,32
NETHERLANDS	11,56	3,89	44,93
Portugal	3,94	3,17	12,51
SPAIN	9,55	3,28	31,30
UK	8,99	6,04	54,27

Sources. Government commitments: EU (2010).
Bank sector total assets (end-2009): ECB (BoE for UK). GDP (2009):Eurostat

The burden of the state interventions as a ratio to GDP is the outcome of two factors: the amounts of public resources committed as a share of the size of the banking sector, and in turn the size of this sector relative to GDP. An interesting question is whether any relationship exists between such two factors. Figure 3 provides a tentative answer, and it is positive. It is easy to see that in those countries where the banking sector is larger in proportion to the economy, Governments have been induced to intervene more heavily in support of the financial system. In particular, the huge amount of funds pledged by Ireland and the UK can be partially explained by the very large size of their financial sectors. The well known "too-big-to-fail" doctrine states that a Government cannot let a very large financial institution go bust, since the implied cost for the whole economic system would be too high. The preliminary evidence reported here points to a sort of "too-big-to-fail" doctrine at the macro level: the incentive for a Government to bail out the financial system altogether is stronger, the larger the size of that sector relative to the economy of the country. Of course, this is an issue deserving further analysis and explanation.

Figure 3 - Government commitments and bank sector size



5.4. Marking-to-Market the bail-out guarantees to the Financial System

We now gather (in Table 5) the risk-neutral figures concerning the probability of a systemic shock to the financial system and those concerning the bail out programs, in order to address the issue of the adequacy of such programs with respect to the actuarial cost of systemic events. In particular, we provide a measure of the liability incurred by the public sector, due to a systemic shock to the financial sector. This exercise rests on the assumption that Governments provide an implicit bail-out guarantee to the financial sector. This assumption is justified both by the huge amount of resources already committed by Governments under the rescue plans illustrated in the previous section, and by the strong correlation between the patterns of CDS spreads between the public and the financial sectors: this evidence shows that the Governments are expected to take up the losses possibly incurred by banks, following a bail-out policy (see Appendix 2).

	Table 5. Mark-to-market of the implicit guarantee to a systemic shock (bn euro)					
	Intensity	**DP**	**LGD**	**Government Liability**	**Commitments**	**Liability - Commitments**
Portugal	6,04%	26,06%	312,12	73,68	20	53,68
Ireland	7,15%	30,05%	980,4	266,85	430	-163,15
Italy	2,65%	12,42%	2248,62	252,98	20	232,98
Greece	12,12%	45,45%	295,14	121,51	28	93,51
Spain	4,73%	21,06%	2068,08	394,57	329	65,57
Germany	0,94%	4,57%	4461,66	184,89	480	-295,11
France	1,36%	6,56%	4594,02	273,00	288,95	-15,95
UK	2,07%	9,85%	5677,2	506,61	444,66	61,95
Netherland	1,70%	8,15%	1330,2	98,23	200	-101,77
Austria	2,79%	13,02%	618,12	72,90	90	-17,10
			Total	2245,23	2330,61	-85,38

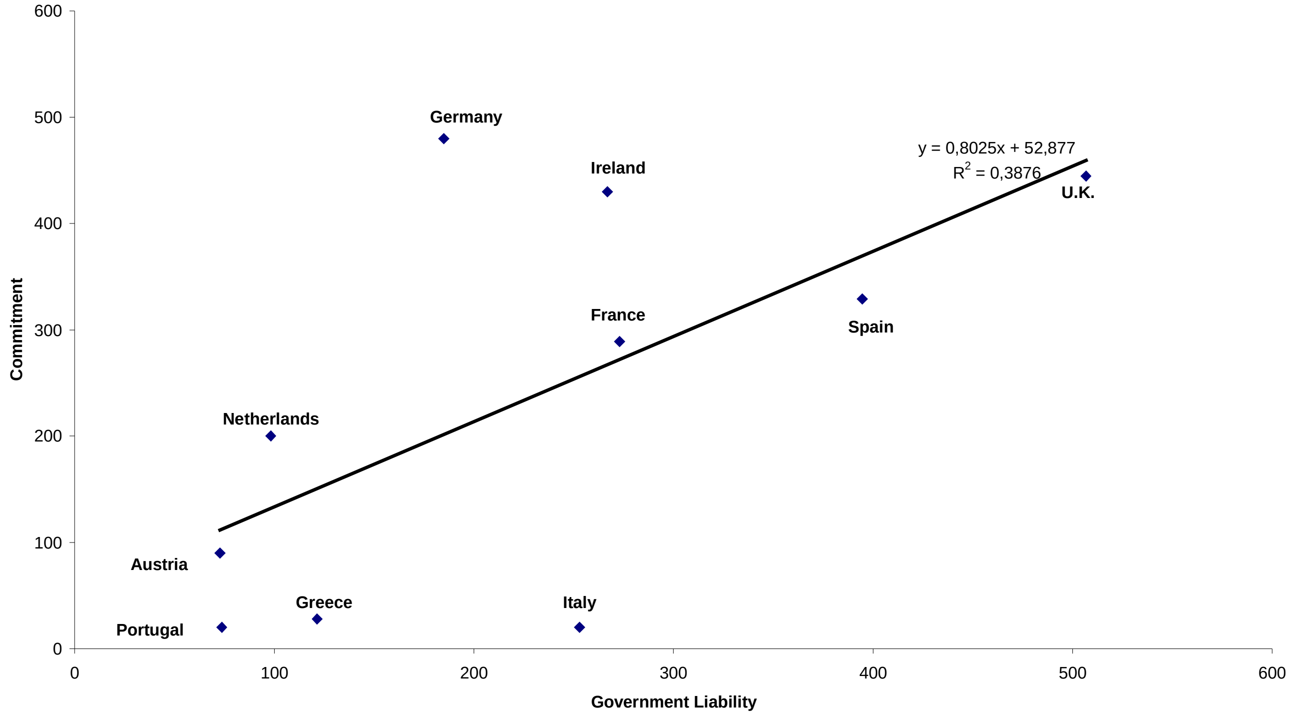
We briefly describe how to read the results of the table. In the first column we report the intensity of a systemic shock evaluated according to our financial stability index on September 15th 2010. In the column headed DP we report the probability of occurrence of the systemic shock over the horizon of our analysis, that is 5 year. Next to it, we report the severity of the systemic shock, computed assuming a *loss given default* figure (LGD) of 60% of the banking sector total assets for each country. We use 60% because this is the value conventionally used to derive quotes in the credit default market. The Government liability is the actuarial value of the implicit bail-out guarantee to the financial sector, and it is computed by multiplying the probability of occurrence of a systemic shock times its severity and the five year discount factor (corresponding to a five year risk free rate of about 2%). The next column reports the commitments to support the banking systems, excluding those spent for individual cases.

As a word of caution we must say that the Government liability is computed under a strong assumption, namely that the Government is called to bail out the whole banking system of the country in case of a default shock. This can be justified by recalling that we are focusing here on systemic risk, which plays a dominant role under the current crisis (see the last column of Table 2). On one hand, it is true that this assumption might lead us to overstate the Government liability. On the other hand, it is also true that the observed loss given default is often larger than what we assume (60%): for example, it is around 80% for Lehman Brothers and Anglo-Irish Bank; under this regard, we might be under-estimating the Government liability in a bail-out intervention.

We can see that the estimated market values of the bail-out guarantees needed to face a systemic crisis in the banking sector are associated to the commitments actually devoted by European Governments to such purpose. On one side, the total amounts are very similar, namely 2245 billion euros against 2330. On the other side, the degree of association can be clearly appreciated in Figure 4. Note that if this association is quite clear across all countries, the difference between guarantees and commitments shows quite a large degree of variation from one country to the others. The two extreme cases are: Germany, for which the commitments are largely higher than the actual value of the guarantee, and this can be associated to the low systemic content of default intensity; and Italy, which reports the lowest level of commitments along with Portugal, in spite of much greater dimension of the banking system. A surprise is that Ireland is the country that, after Germany, has the highest positive difference between the value of commitments and that of the guarantees[3].

[3] We have run the exercise shown in Figure 4 also by subtracting the exposure to the Government of the own country from the total assets of each bank, since in several countries this exposure is quite relevant. In case of distress, a bank can transfer its portfolio of Government bonds to its creditors, thus reducing the liability of the Government involved in a bail-out. However the results obtained are very similar to those reported here, so we decided not to show them to save space (of course, they are available upon request).

Figure 4. Government liabilities due to systemic shocks and actual commitments



Finally, in Table 6 we report the impact of the bail-out guarantee on the public debt/GDP figures. We can see that the problem is worst for Ireland, and the reason is the huge dimension of the banking system relative to GDP (see Table 4, column B). The impact is lower but still quite relevant for the other PIIGS; among these countries, Italy is the one which suffers the lowest burden, thanks to the relative strength of its banking system. Among the other countries, UK is the one most hit by the bail-out liability: just like it happens for Ireland, the size of the banking sector plays a crucial role.

	Table 6. Bail-out Government liability and Debt/GDP		
	Debt/GDP	**Liability/GDP**	**Total**
Portugal	76,80%	44,96%	121,76%
Ireland	64,00%	163,17%	227,17%
Italy	115,80%	16,63%	132,43%
Greece	115,10%	51,16%	166,26%
Spain	53,20%	37,54%	90,74%
Germany	73,20%	7,68%	80,88%
France	77,60%	14,31%	91,91%
UK	68,10%	32,34%	100,44%
Netherlands	60,90%	17,23%	78,13%
Austria	66,50%	26,33%	92,83%

6. Summary and conclusions

We have introduced a new methodology for measuring the systemic risk of default of the banking sector and we have applied it to the European countries. Our methodology is built within the framework of multivariate intensity based models, and it is based on a homogeneous version of such models, called Cuadras – Augé distribution. The filter we derive measures the relevance of systemic risk (due to the likelihood of a shock spreading to the whole banking system) relative to the average default probability of individual banks. We have computed the index for 10 European countries, exploiting the information incorporated in the CDS premia of 44 banks over the period January 2007 – September 2010. In this way, we can provide a market based measure of the liability incurred by the Governments, due to the implicit bail-out guarantees they provide to the financial sector of the economy. We then compare this estimate with the actual resources committed so far by the European Governments through the rescue plans adopted between October 2008 and March 2010.

Our main results may be summarized as follows. During the financial crisis, the systemic component of the default risk in the banking sector has significantly increased in all countries, with the exception of Germany and the Netherlands. The most recent data (September 2010) show that almost all the risk of default of individual banks is accounted for by the systemic component. As a consequence, the Governments' liability implicit in the bail out guarantee amounts to a quite relevant share of GDP in several countries: it is huge for Ireland, lower but still important for the other PIIGS (Italy is the least affected within this group) and for the UK. The two crucial factors determining these results are: (i) the correlation among the default probabilities of banks within a country; (ii) the size of the banking system relative to the economy of each country. Finally, our estimate is very close to the overall amount of money committed in rescue plans at the European level. However, strong cross-country differences emerge under this regard: in particular, Germany and Ireland seem to have committed an amount of resources much larger than needed; to the contrary, the Italian Government has committed much less than it should.

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Appendix 1. Cuadras-Augé Filter

Assume we want to represent a set of variables with Marshall-Olkin distribution with a set of homogeneous variables. Homogeneous means that the variables must have:

a) same marginal distribution
b) same bivariate correlation

We design here a filter to accomplish this task. Since the filter turns the Marshall-Olkin system into a homogeneous one, we call the technique Cuadras-Augé filter because it turns the system into a homogeneous (and so exchangeable) Marshall-Olkin system that is known as Cuadras-Augé system.

Let us denote ρ_{ij} the Pearson correlation statistics between the *i*-th and *j*-th variables, and we know that

$$\rho_{ij} = \frac{\lambda_{ij}}{\hat{\lambda}_i + \hat{\lambda}_j - \lambda_{ij}} \tag{A1}$$

where $\hat{\lambda}_i$ and $\hat{\lambda}_j$ denote marignal intensities and λ_{ij} collects common shocks that reach both assets. Remember that in Marshall-Olkin model this is also equal to the Kendall's *tau* statistic. We want to design a system, as close as possible to the original system with

$$\rho = \frac{\overline{\lambda}}{2\hat{\lambda} + \overline{\lambda}} \tag{A2}$$

for all the pairs. The straightforward choice for marginal intensities is to set them equal to the average, namely

$$\hat{\lambda} = \hat{\lambda}_m = \frac{\sum_{i=1}^{n} \hat{\lambda}_i}{n} \tag{A3}$$

As for the common intensity, a natural choice would be to recover it from some average correlation value: in other terms, we want to extract $\overline{\lambda}$ from an average ρ value. To obtain this result, it is natural to work with harmonic mean. We remind that this mean is defined as

$$H = \frac{k}{\sum_{i=1}^{k} \frac{1}{x_i}} \tag{A4}$$

for a set of k positive variables x_i. Let us then work with the inverse of ρ_{ij}.

$$\frac{1}{H} = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{1}{\rho_{ij}} \tag{A5}$$

Notice that the sum includes $n(n - 1)/2$ terms, that is the number of elements below the diagonal of the correlation matrix. We now substitute the correlation terms

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{1}{\rho_{ij}} = \sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \left(\frac{\hat{\lambda}_i + \hat{\lambda}_j}{\lambda_{ij}} - 1 \right) \tag{A6}$$

If we substitute $\overline{\lambda}$ for λ_{ij} and we consider the number of elements in the sum we obtain

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{1}{\rho_{ij}} = \sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{\hat{\lambda}_i + \hat{\lambda}_j}{\overline{\lambda}} - \frac{n(n-1)}{2} \tag{A7}$$

Notice that in the double sum each marginal intensity $\hat{\lambda}_i$ compares $n - 1$ times. So, we have

$$\sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{1}{\rho_{ij}} = (n-1) \sum_{i=1}^{n} \frac{\hat{\lambda}_i}{\overline{\lambda}} - \frac{n(n-1)}{2} = n(n-1) \frac{\hat{\lambda}_m}{\overline{\lambda}} - \frac{n(n-1)}{2} \tag{A8}$$

where we have substituted the average marginal intensity. If we now substitute equation (A8) in (A5) we have

$$\frac{1}{H} = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n-1} \frac{1}{\rho_{ij}} = 2 \frac{\hat{\lambda}_m}{\overline{\lambda}} - 1 \tag{A9}$$

Solving for $\overline{\lambda}$ we obtain

$$\overline{\lambda} = \frac{2}{1 + \frac{1}{H}} \hat{\lambda}_m \tag{A10}$$

Using the same strategy it is possible to design a filter that uses rank-correlation, that is Spearman's *rho*. In this case one exploits the relationship

$$\rho_{s,ij} = \frac{3\lambda_{ij}}{2\left(\hat{\lambda}_i + \hat{\lambda}_j\right) - \lambda_{ij}} \tag{A10}$$

where $\rho_{s,ij}$ denotes rank correlation. Doing the math we obtain

$$\overline{\lambda} = \frac{4}{3} \frac{1}{\frac{1}{3} + \frac{1}{H_s}} \hat{\lambda}_m \tag{A11}$$

where H_s is the harmonic mean of rank correlation.

Appendix 2. The data set and individual country information

Table A.1 below provides a description of our data set, which includes 44 banks representing 10 European countries.

Table A1. Our sample

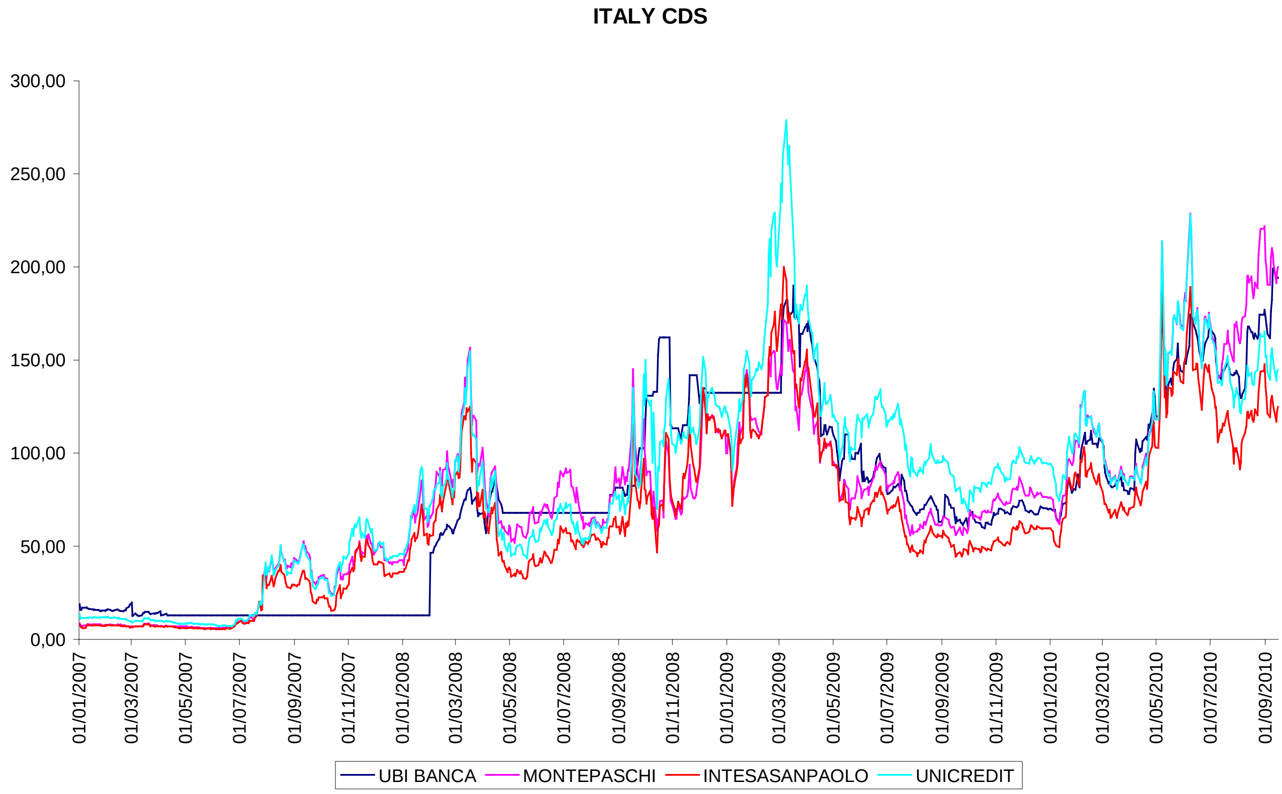
	Assets (bn euro) (A)	Sample banks' total assets over bank sector assets (%) (B)	Risk-weighted assets (bn euro) (C)	Exposure to own country Government (% of assets) (D)	Leverage (assets/equity) (E)	Tier 1 ratio (F)
ALLIED IRISH BANKS	179,02		121,6	2,31	14,6	7,0
BANK OF IRELAND	181,82		104,6	0,65	14,0	9,2
IRELAND	**360,84**	**22,1**	**226,2**	**1,48**	**14,3**	**8,1**
PIRAEUS BANK	48,95		37,4	16,97	13,6	9,1
ALPHA BANK	67,83		51,1	7,47	10,1	11,6
EFG EUROBANK	84,62		47,6	8,81	14,3	11,2
NATIONAL BANK GREECE	113,99		67,4	17,33	15,0	11,3
GREECE	**315,38**	**64,1**	**203,5**	**12,65**	**13,3**	**10,8**
BANCO PASTOR	32,17		18,7	8,20	14,0	10,5
BANKINTER	54,55		30,7	3,18	17,0	7,5
CAIXA CATALUNA	273,43		52,9	1,49	50,6	6,6
SABADEL	83,22		58,0	5,85	13,6	9,0
BANCO POPULAR	129,37		92,6	5,85	14,5	9,1
CAJA MADRID	193,01		223,1	12,55	7,6	8,6
BBVA	394,41		290,1	13,22	10,0	9,4
SANTANDER	423,08		562,6	11,97	5,2	10,0
SPAIN	**1583,22**	**45,9**	**1328,5**	**7,79**	**16,6**	**8,8**
BANCO BPI	47,55		26,1	8,88	17,0	8,5
ESPIRITO SANTO	85,31		67,9	5,49	11,7	7,7
BCP	13,99		65,6	6,81	1,9	9,3
CAIXA GERAL	121,68		71,0	5,56	13,7	8,4
PORTUGAL	**268,53**	**51,6**	**230,6**	**6,69**	**11,1**	**8,5**
UBI BANCA			85,7			8,0
MONTEPASCHI	225,87		120,9	12,29	15,7	7,5
INTESASANPAOLO	424,48		361,8	15,00	9,9	8,3
UNICREDIT	375,52		452,4	10,34	6,9	8,6
ITALY	**1025,87**	**27,4**	**1020,7**	**12,54**	**10,8**	**8,1**
SNS BANK	58,74		25,9	6,00	22,6	10,7
ABN AMRO	203,50		118,7	4,83	9,8	13,0
RABOBANK	425,87		236,3	2,21	12,2	14,1
ING	888,11		332,4	0,47	19,9	10,2
NETHERLAND	**1576,22**	**71,1**	**713,3**	**3,38**	**16,1**	**12,0**
LLOYDS	648,25		580,4	0,93	9,0	9,6
BARCLAYS	1561,54		450,2	0,69	20,9	13,0
HSBC	851,05		871,7	0,00	7,1	10,8
RBS	1311,89		515,7	1,79	15,6	14,4
UK	**4372,73**	**46,2**	**2418,0**	**0,85**	**13,2**	**12,0**
BAYERN LB			135,7			10,9
LBBW			142,5			9,8
WEST LB			35,7			14,4
HSH NORDBANK	171,33		71,4	6,75	14,9	10,5
POSTBANK	227,97		68,7		34,0	7,1

DZ BANK	219,58		95,0		19,3	9,9
HYPO	362,24		81,0		41,6	9,4
COMMERZ	609,79		280,1	7,02	14,7	10,5
DEUTSCHE BANK	1542,66		273,5		40,7	12,7
GERMANY	**3133,57**	**42,1**	**1183,5**	**6,88**	**27,5**	**10,0**
SOCIETE GENERALE	891,61		324,1	1,69	21,2	10,7
CREDIT AGRICOLE	522,38		538,9	4,86	8,6	9,7
BNP	1246,15		620,7	1,45	14,1	10,1
FRANCE	2660,14	**34,7**	**1483,7**	**2,67**	**14,7**	**10,2**
RZB	148,95		94,5	2,78	12,1	9,3
ERSTE BANK	79,72		125,5	7,27	5,0	9,2
AUSTRIA	**228,67**	**22,2**	**219,9**	**5,03**	**8,6**	**9,3**

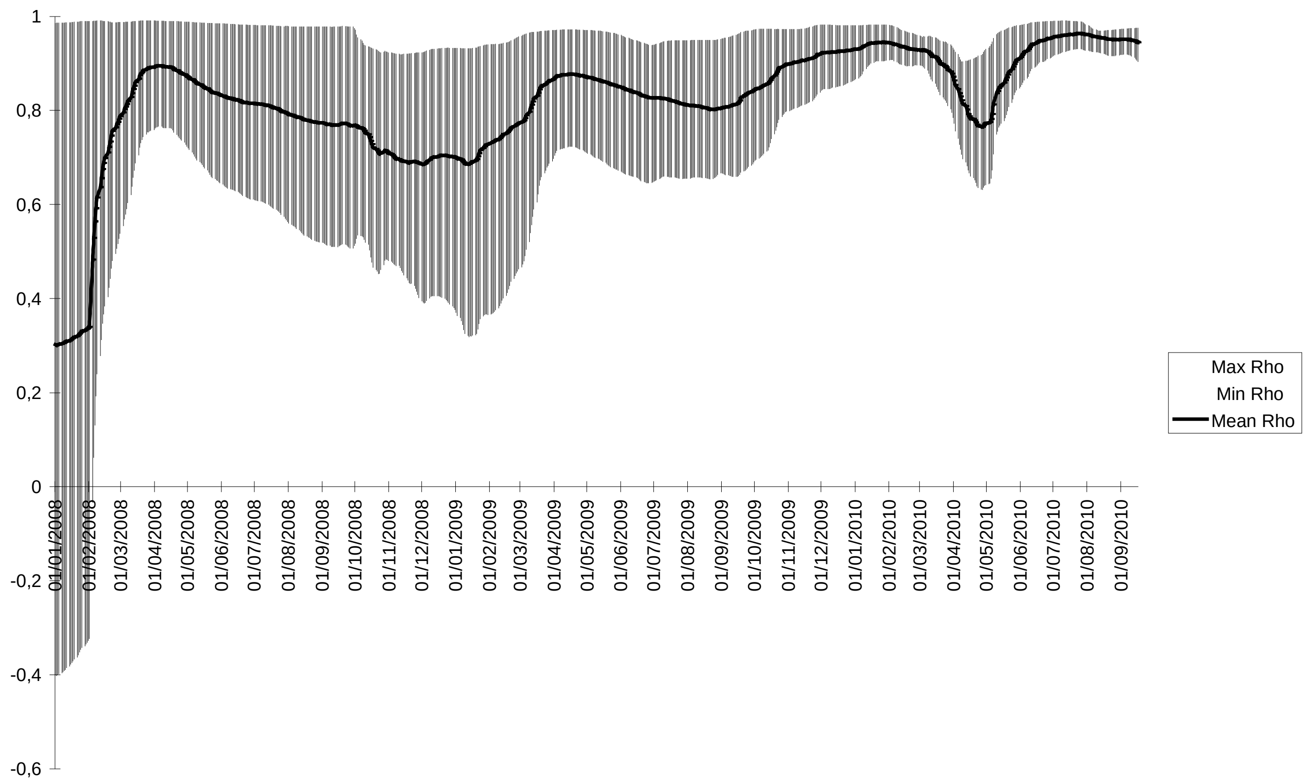
Country lines show total values for columns (A)-(B)-(C) and mean values for columns (D-(E)-(F)

Sources. Sample banks' assets: Bankscope. Bank sector total assets: ECB (BoE for UK).Other items: Unicredit (2010) and our own computations.

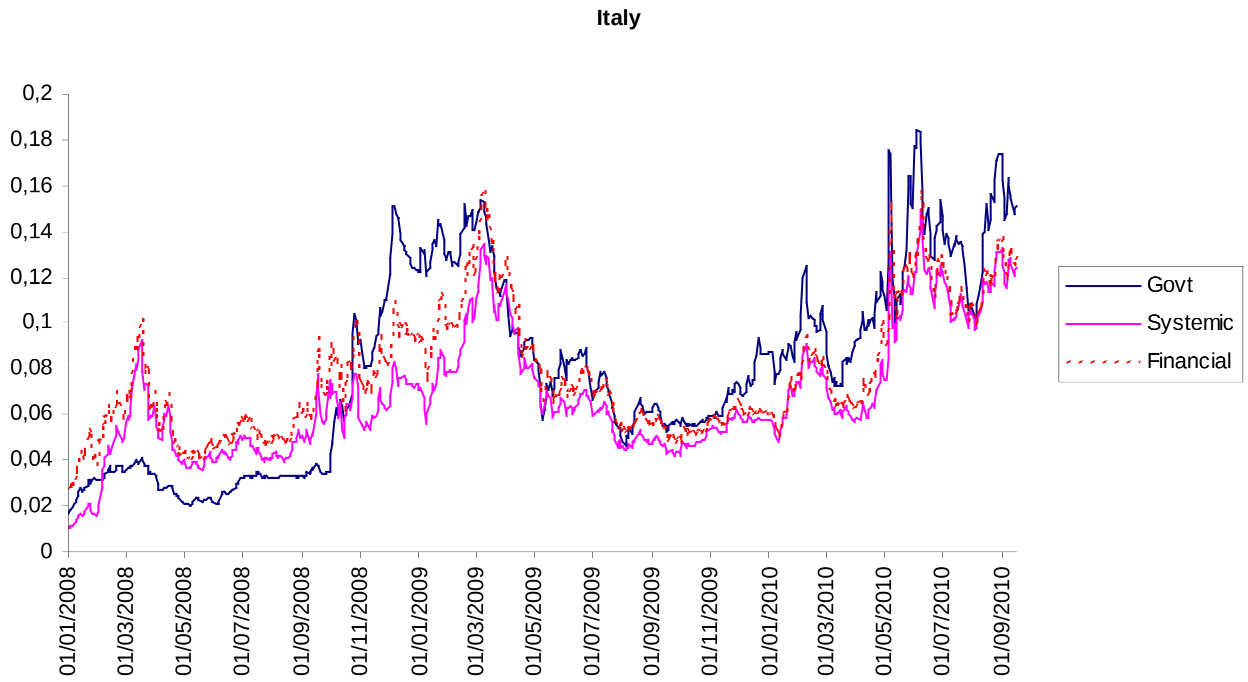
In the following, we report for each country: (i) the time series of CDS premia for each bank included in our sample; (ii) the rank correlation among the survival probabilities of those banks; (iii) the systemic and average default probabilities of the financial sector, as well as that of the Government, over a five year horizon. Abstracting from specific features of individual countries, some general patterns emerge from the pictures below. The bank CDS premia closely reflect the evolution of the financial crisis over the four years 2007 – 2010. More interestingly, we can observe a significant increase of the mean correlation of survival probabilities in the most recent time interval, together with an increasing concentration of bivariate correlation figures around the mean. Finally, the third graph for each country shows a remarkable co-movement of default probabilities between the financial sector and the Government sector, pointing to a strong impact of the troubles hitting the banking system on the sustainability of public debt. This evidence supports the assumption that the Governments are expected to take up the losses possibly incurred by banks, following a bail-out policy.



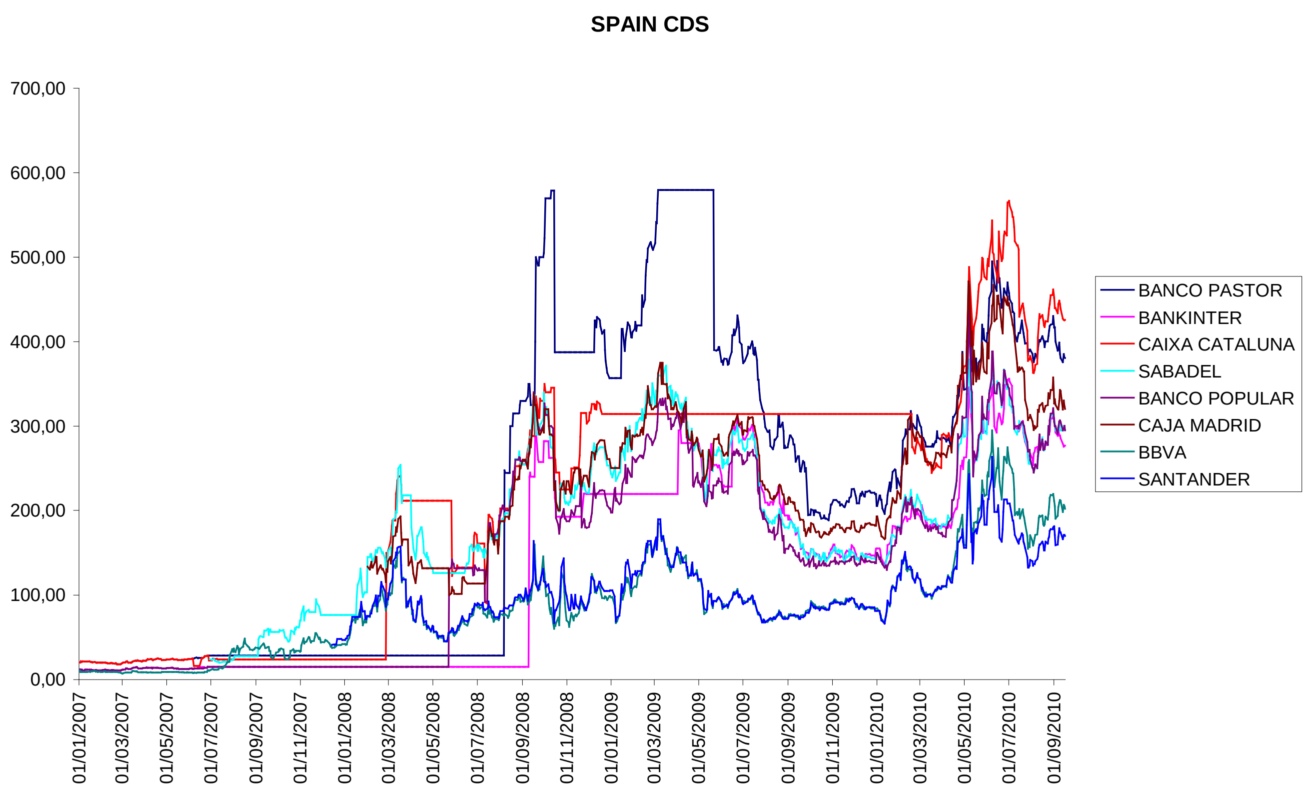
CDS premia. Five year maturity. Italy.



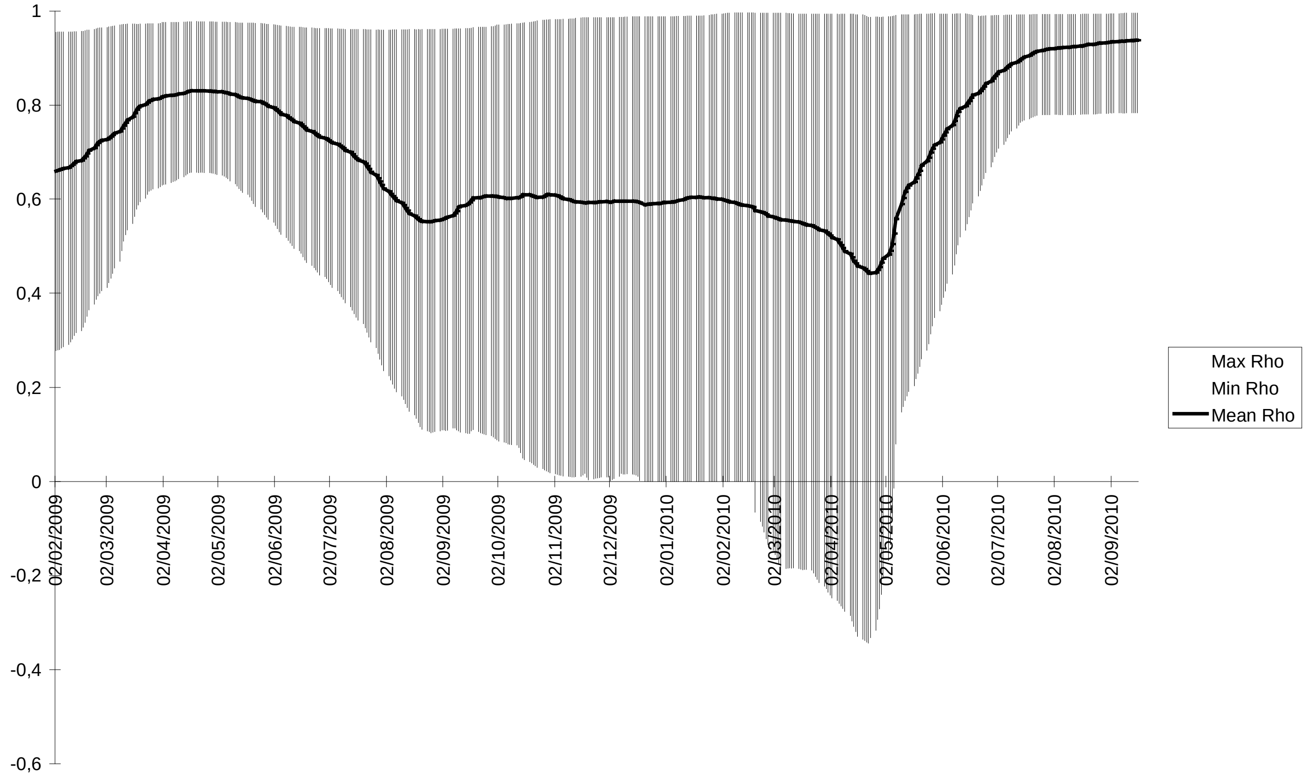
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. Italy.



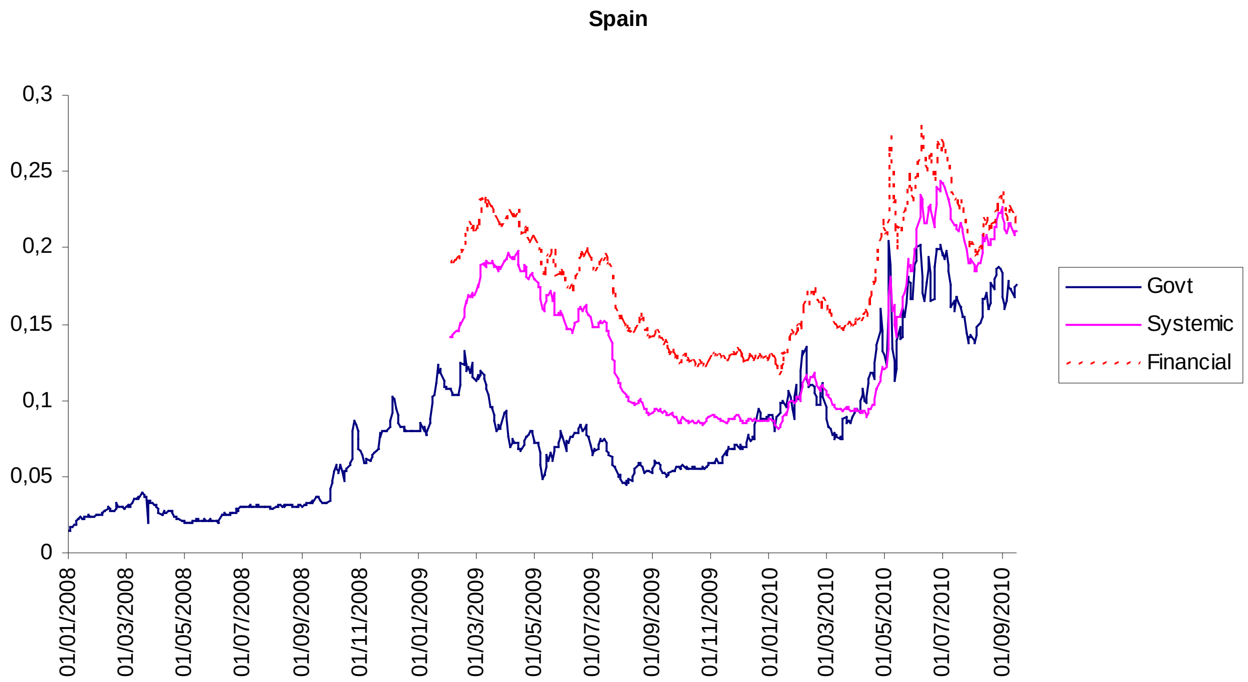
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Italy.



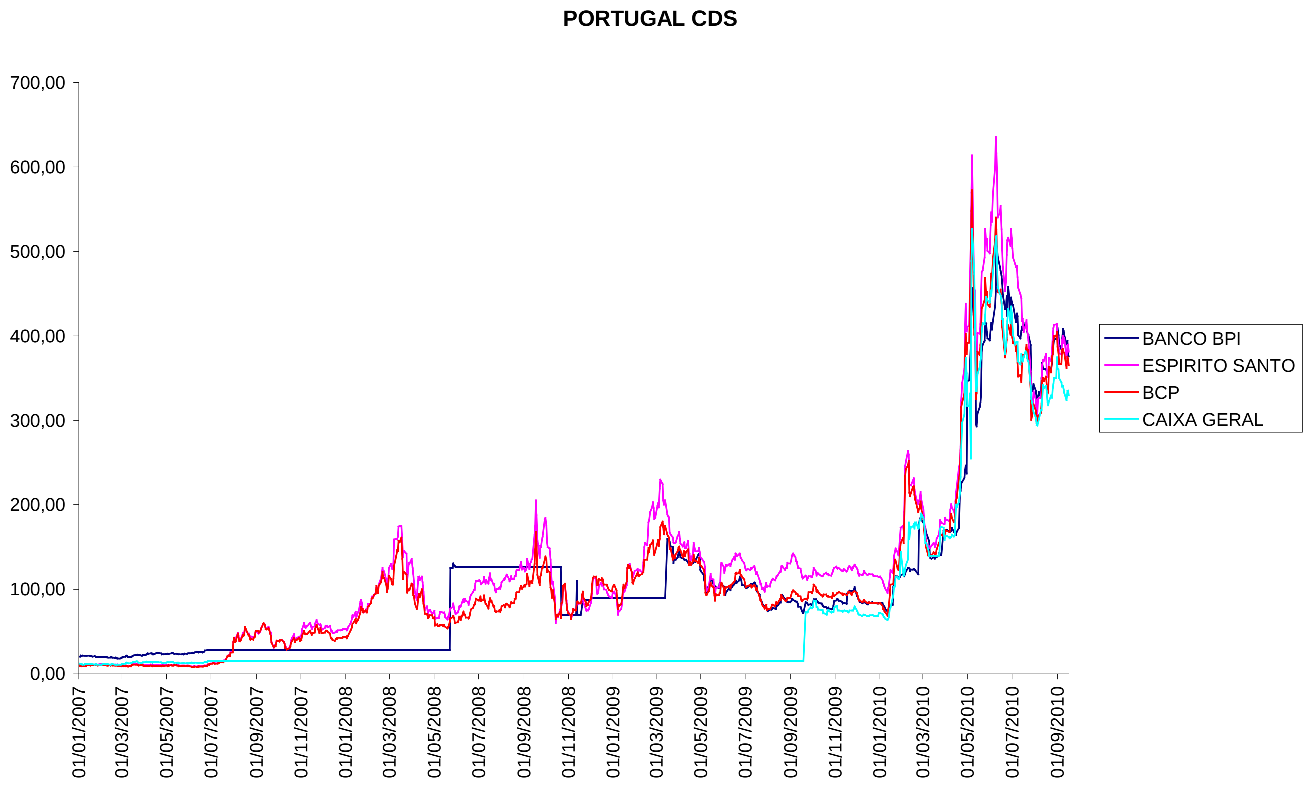
CDS premia. Five year maturity. Spain.



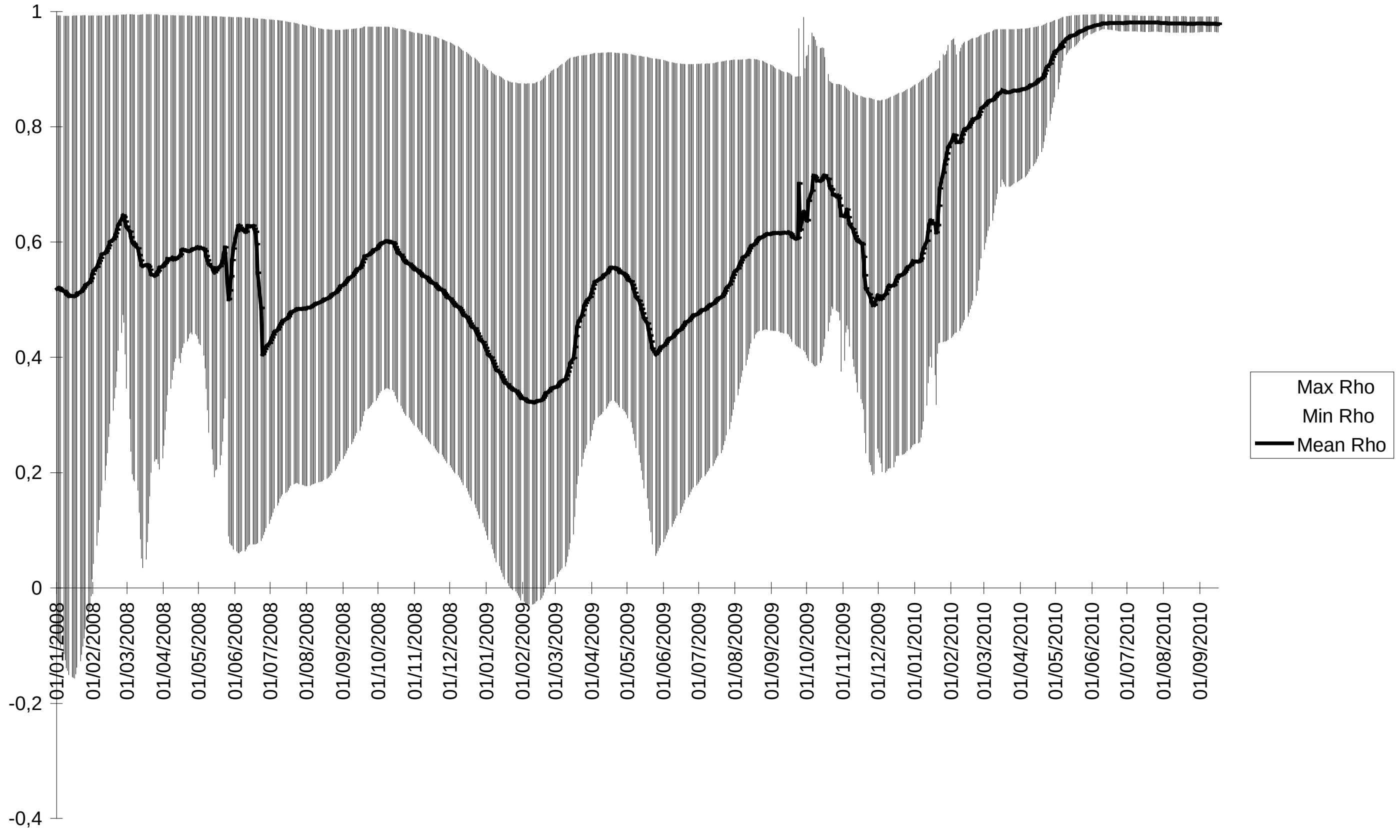
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. Spain.



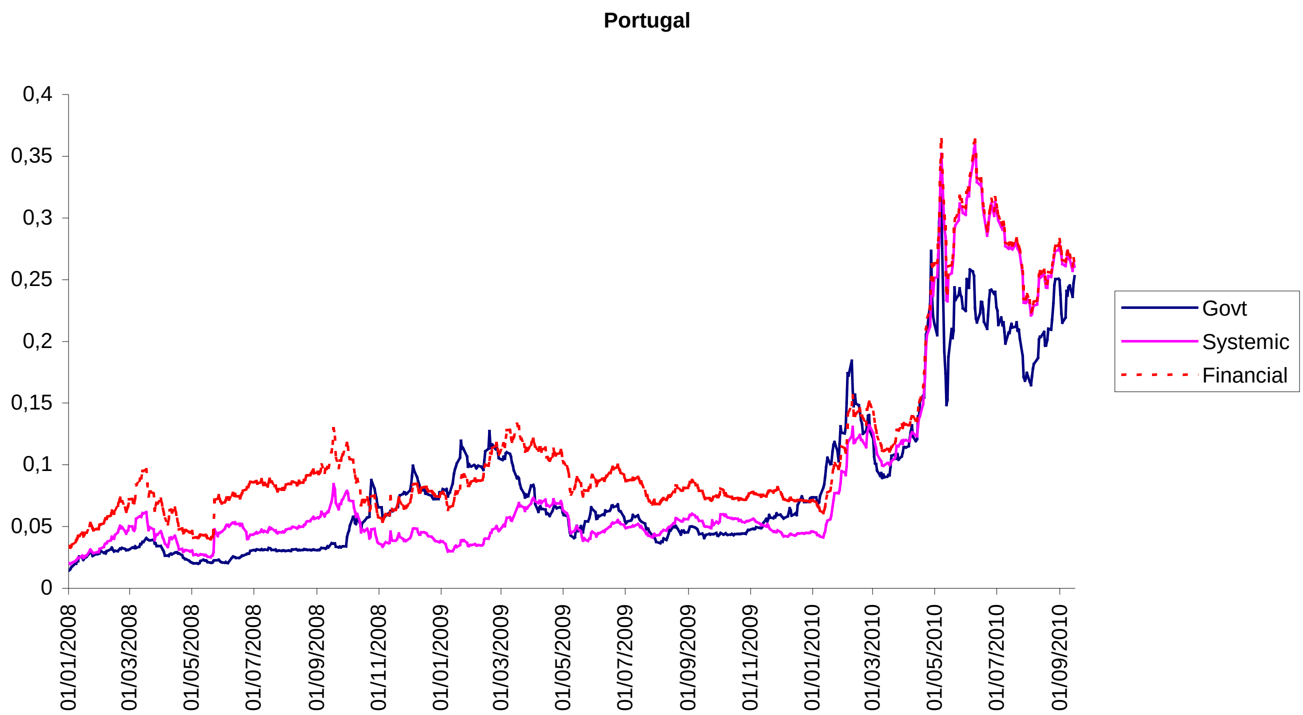
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Spain.



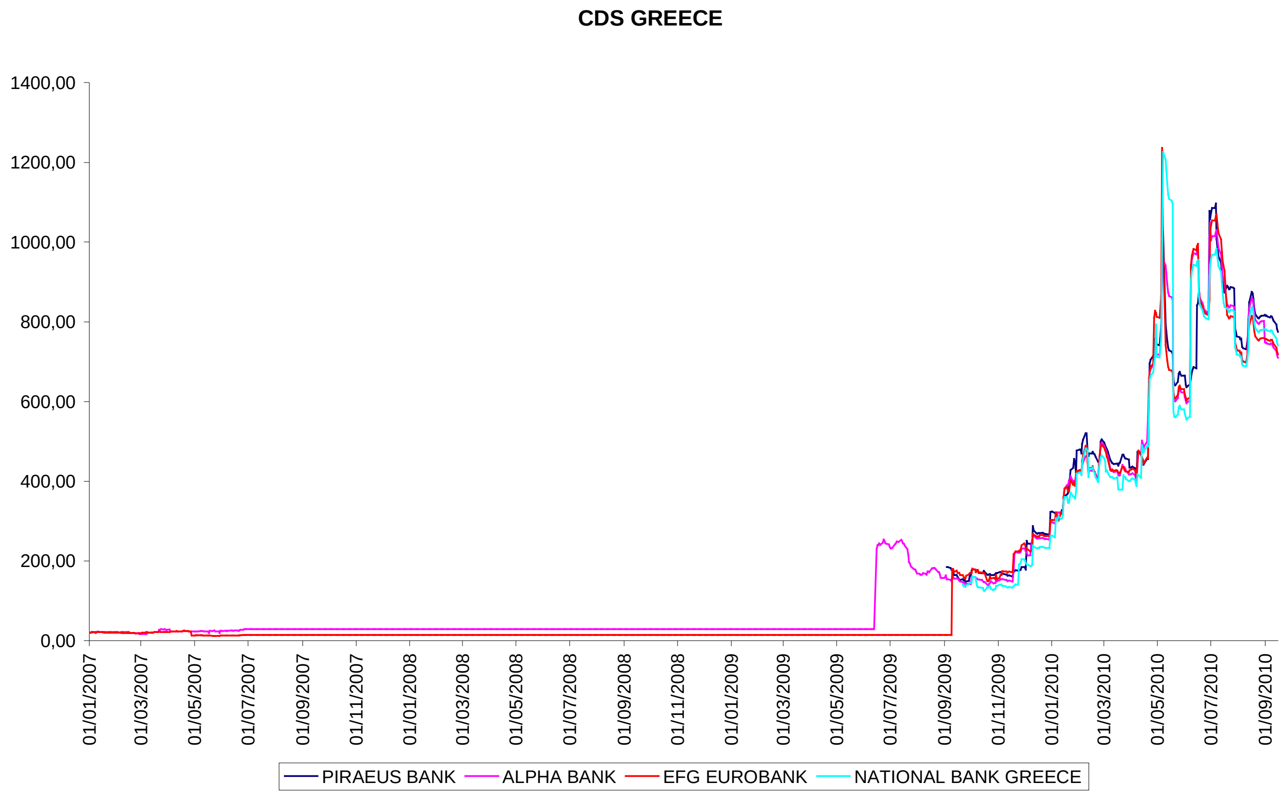
CDS premia. Five year maturity. Portugal



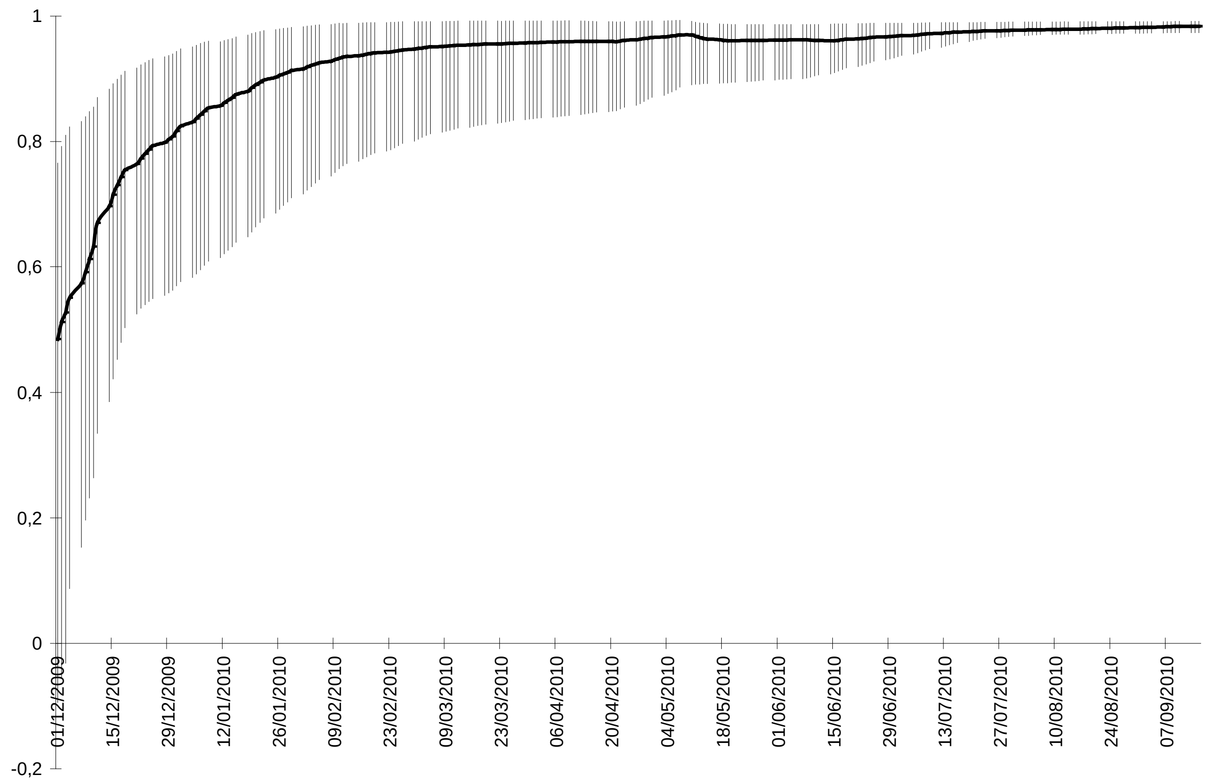
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. Portugal



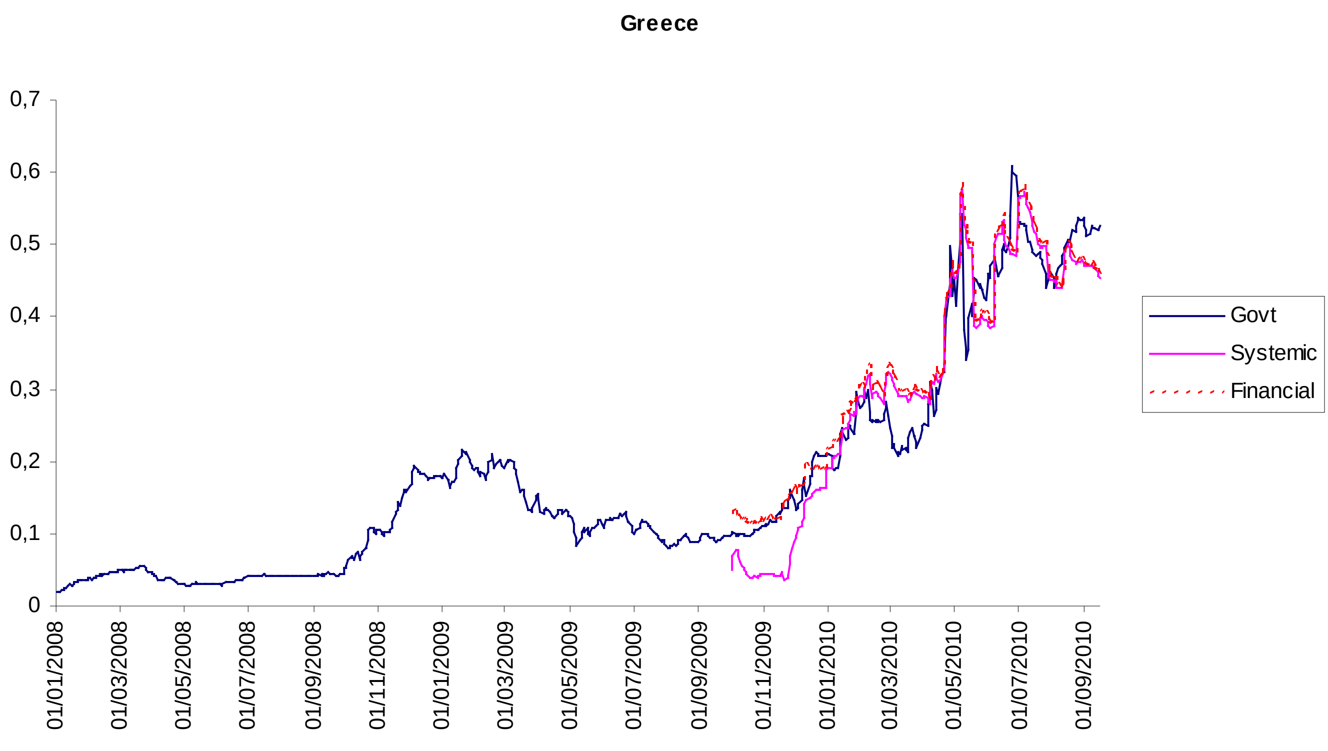
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Portugal.



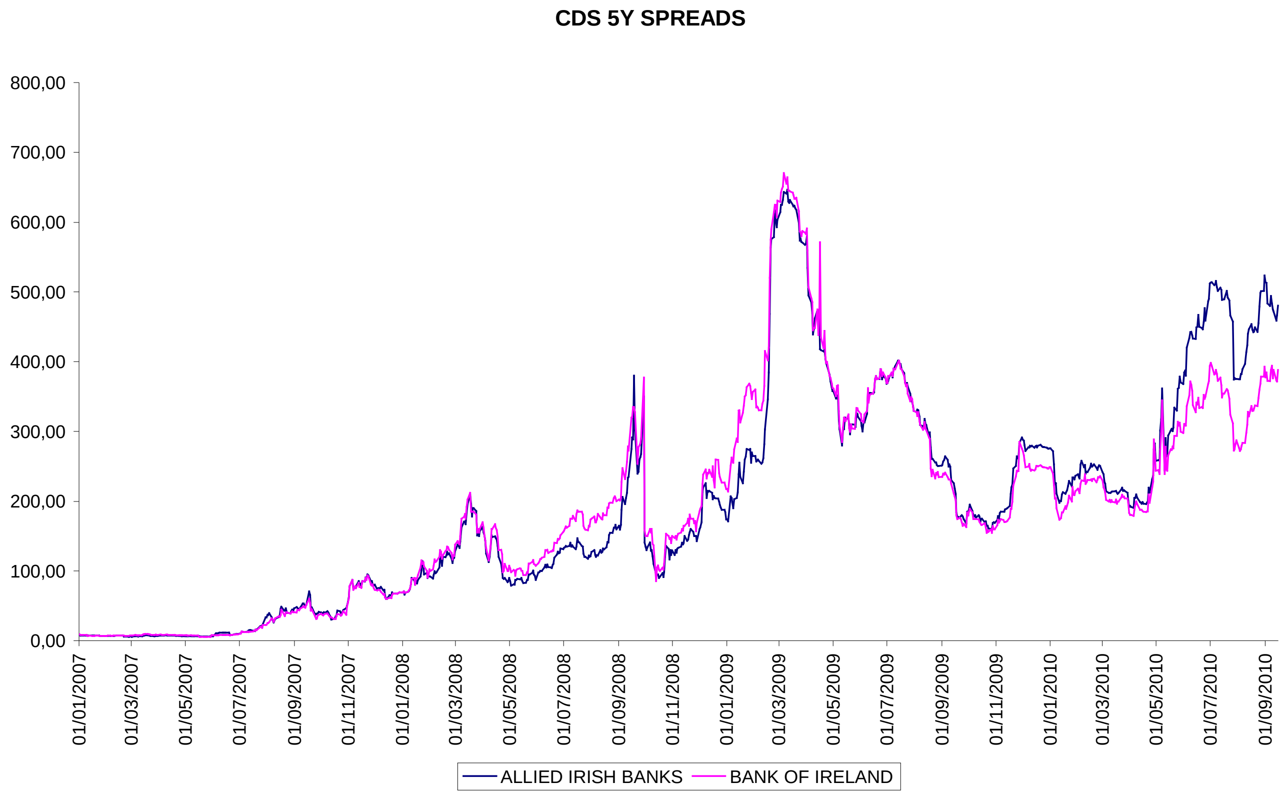
CDS premia. Five year maturity. Greece



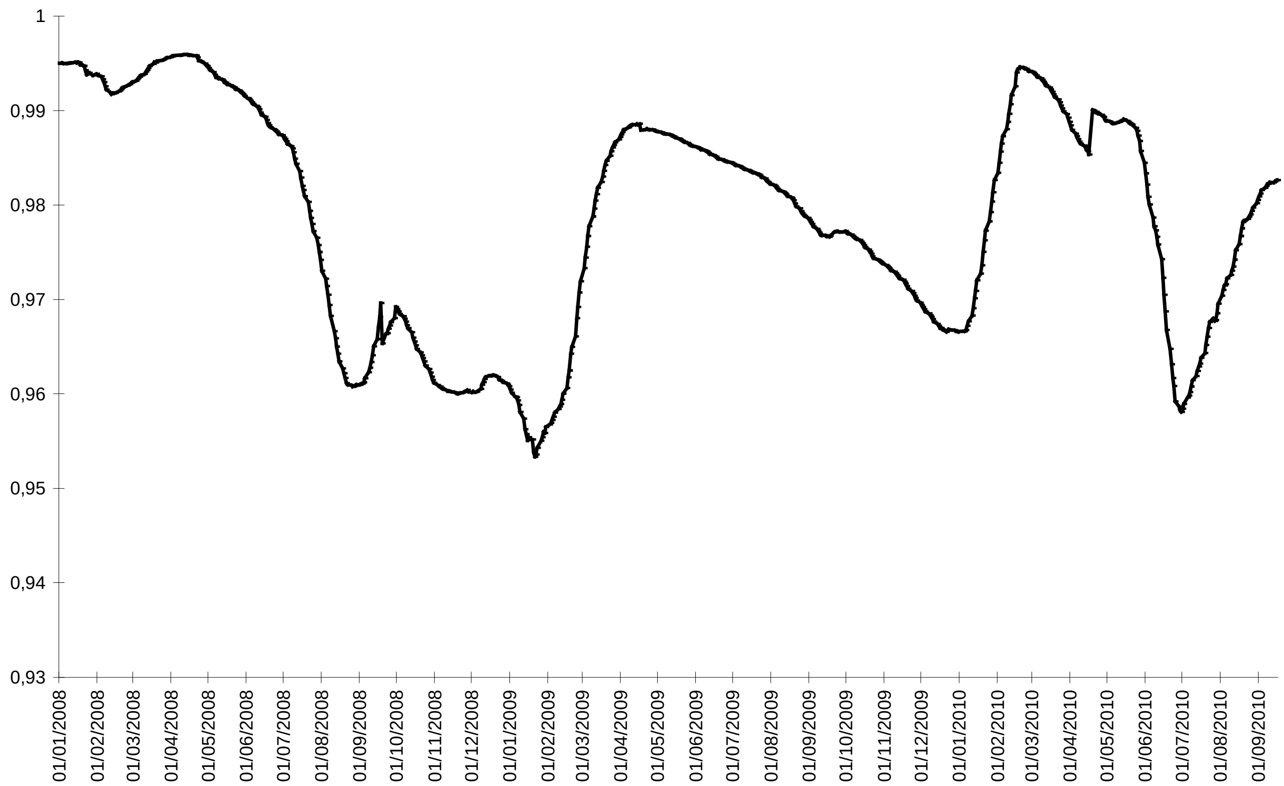
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. Greece.



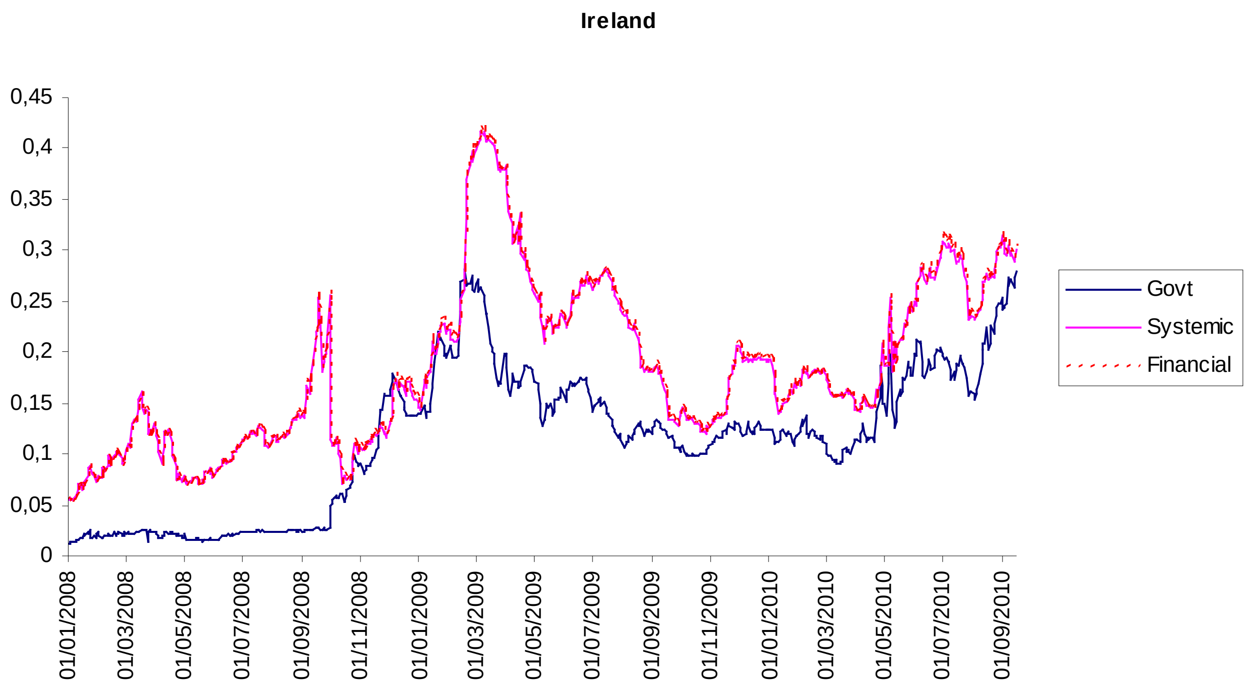
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Greece.



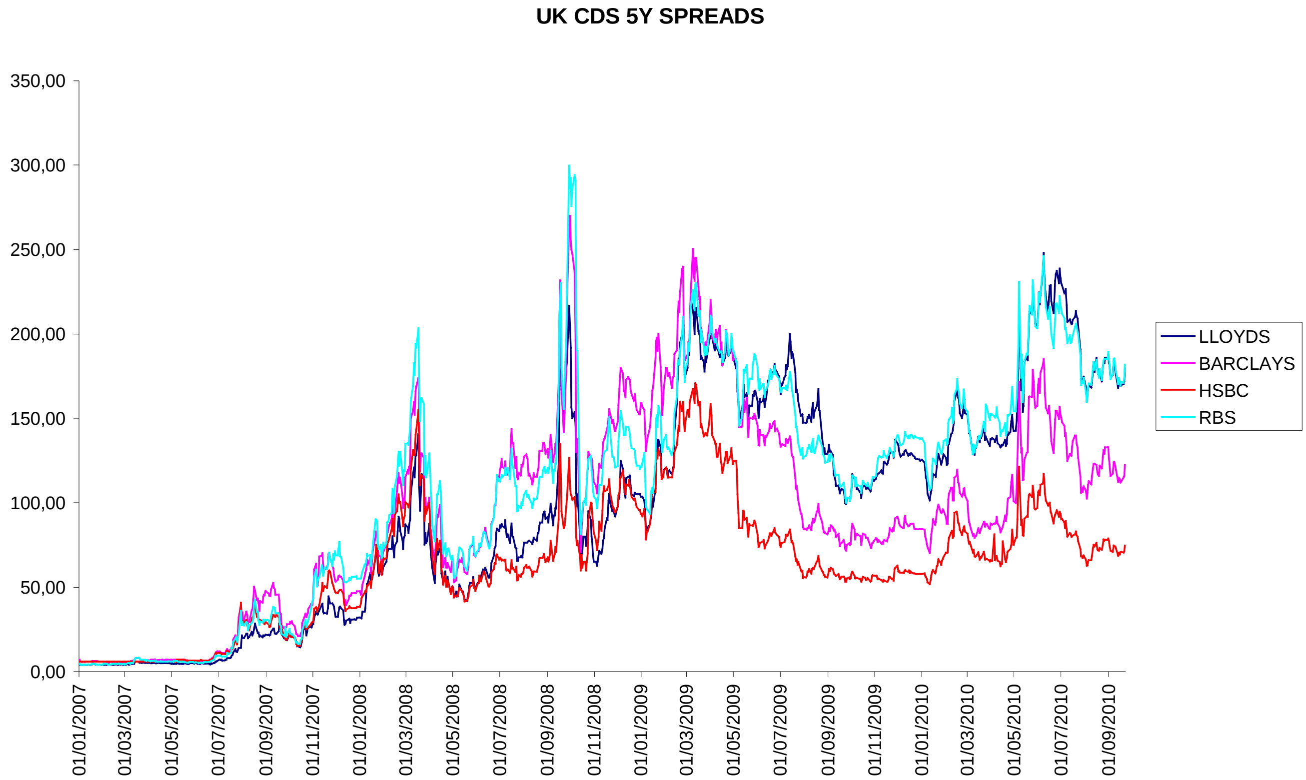
CDS premia. Five year maturity. Ireland



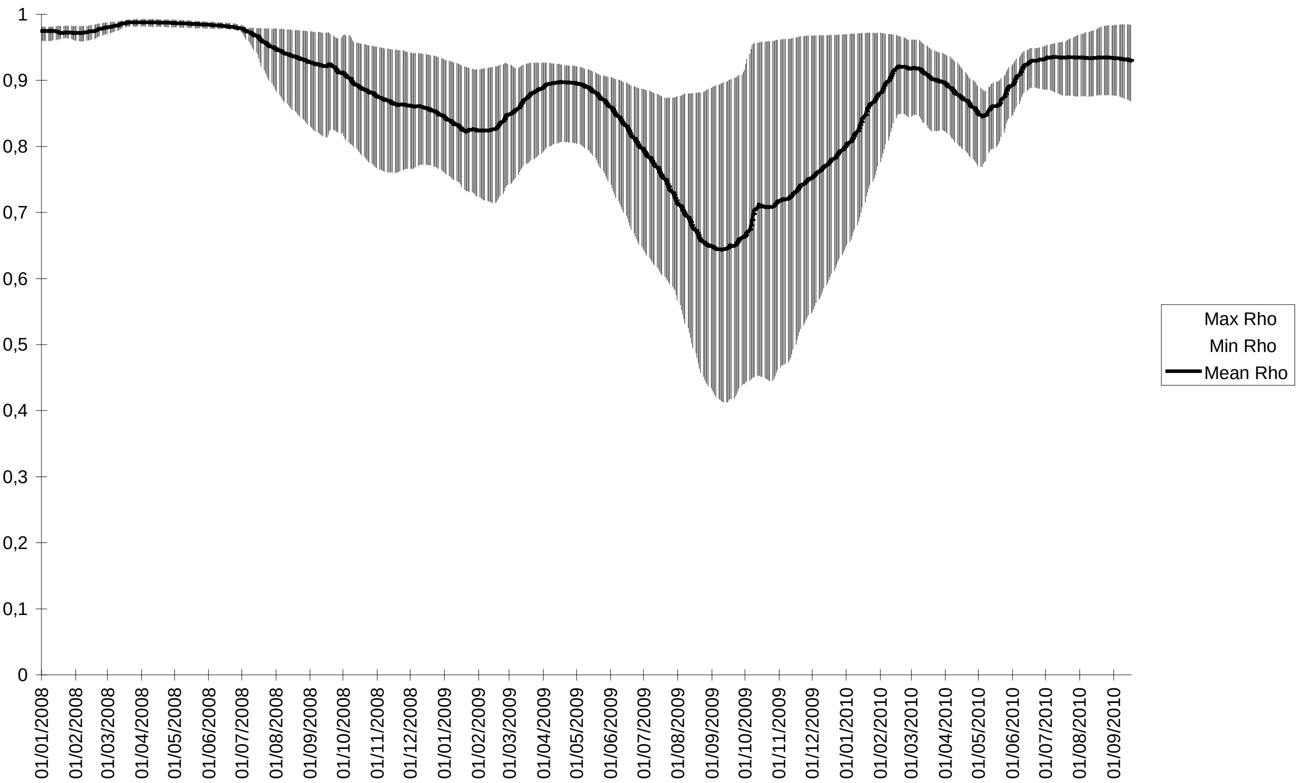
Rank correlation of survival probability. 12 month moving window. Ireland.



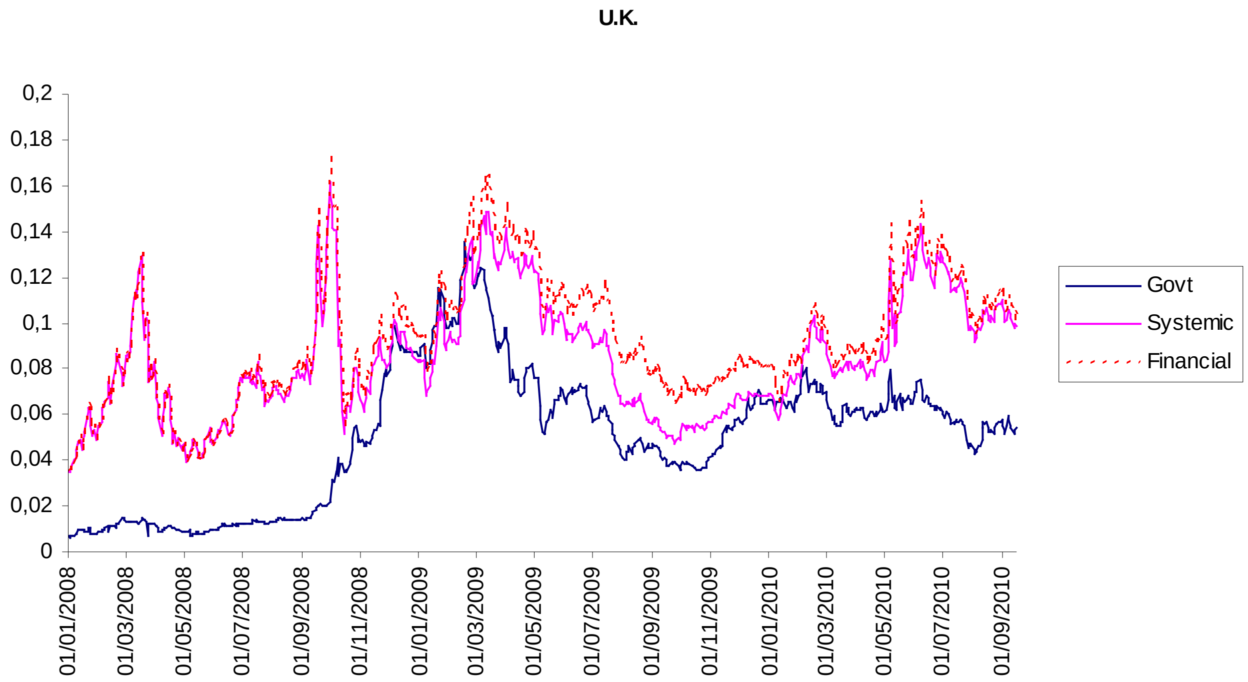
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Ireland.



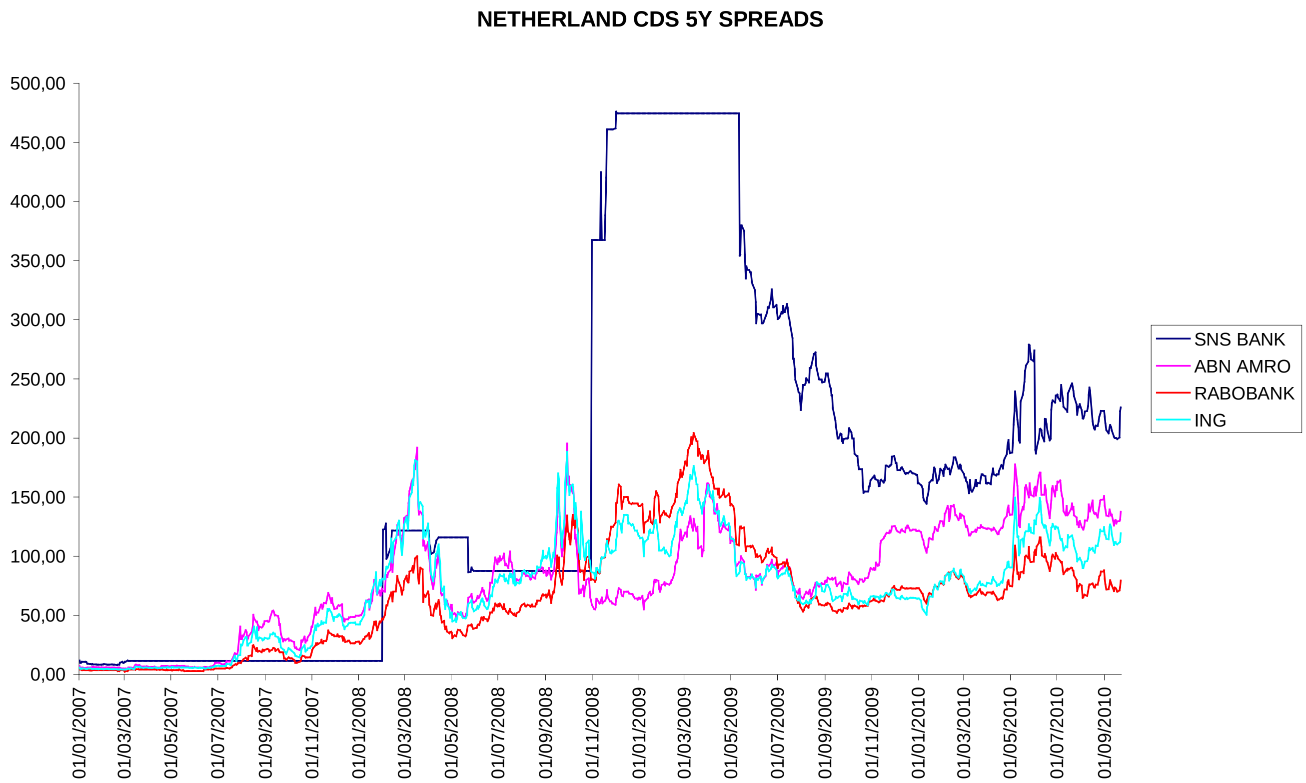
CDS premia. Five year maturity. UK



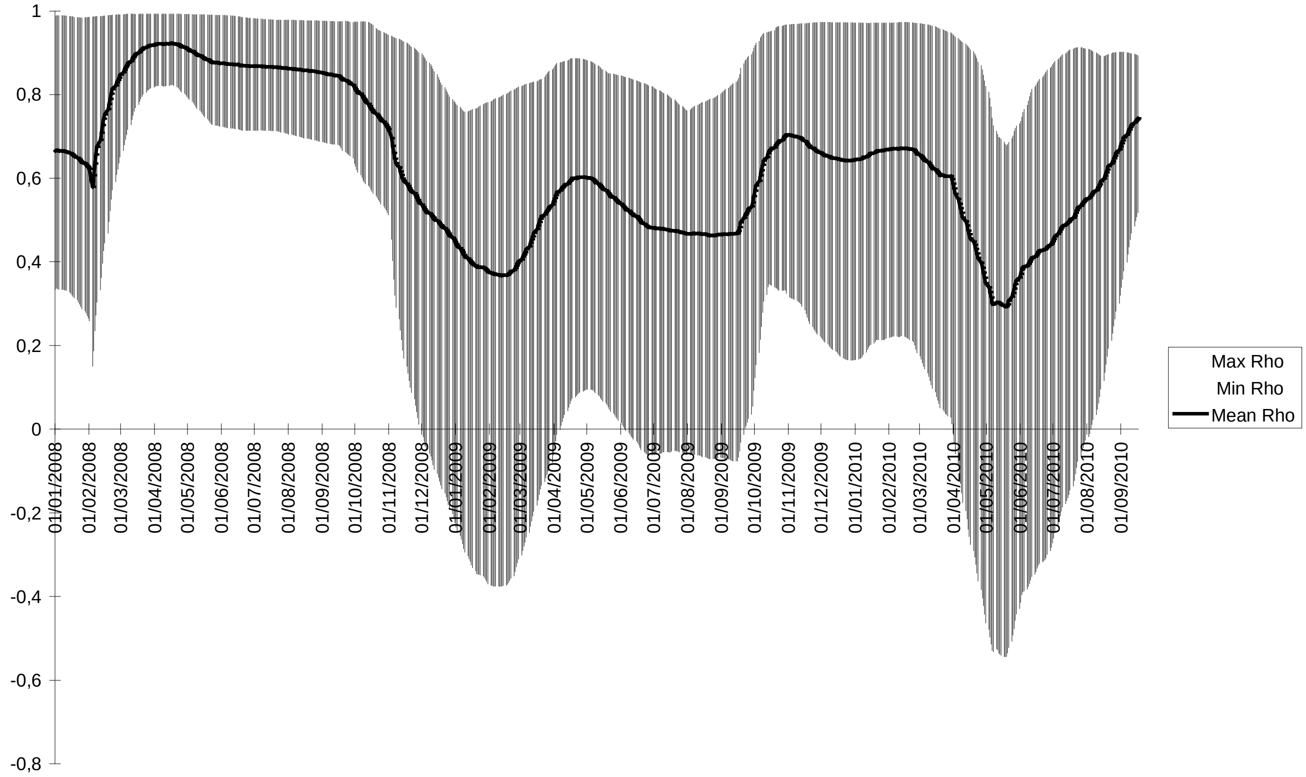
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. UK



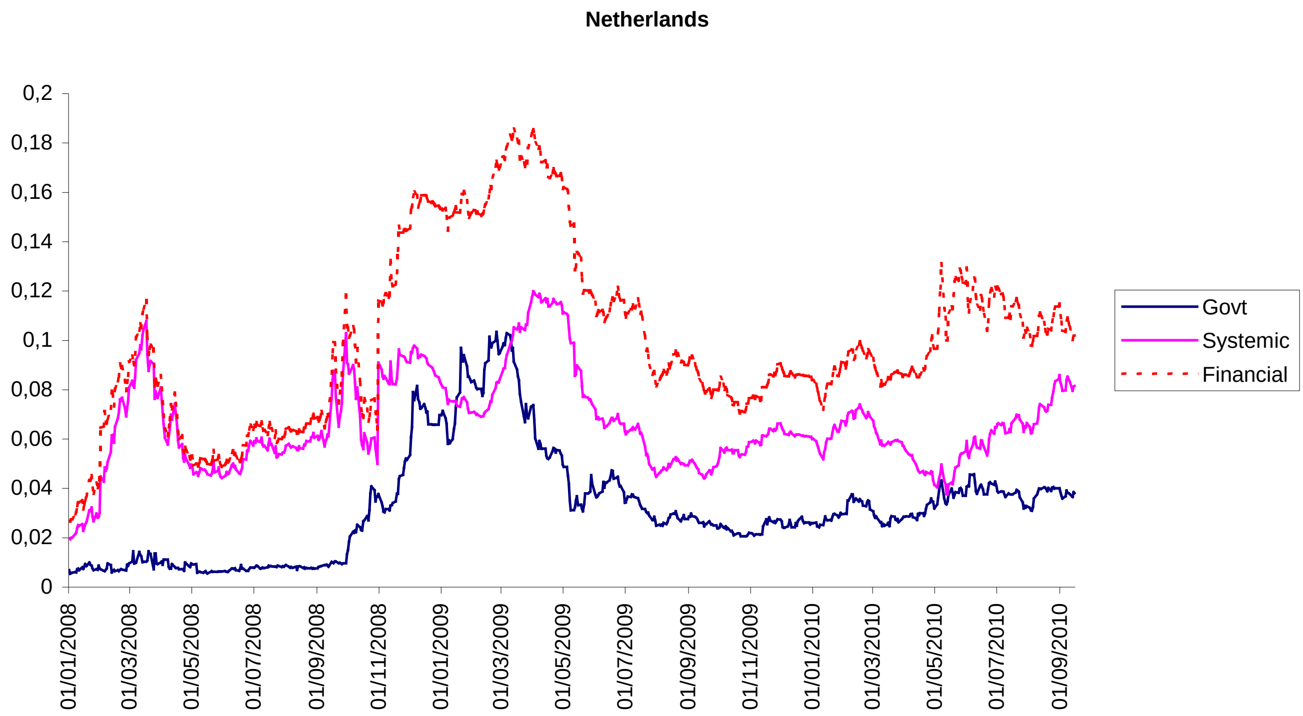
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. UK



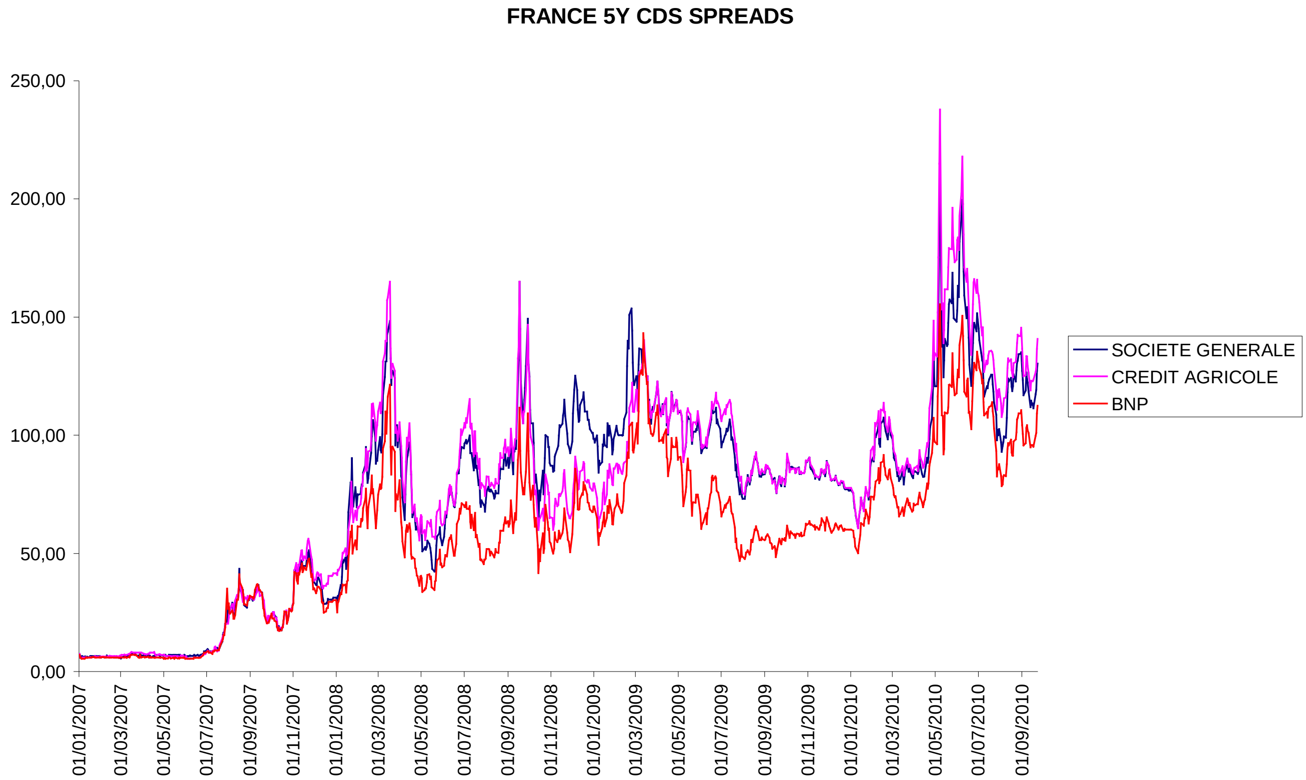
CDS premia. Five year maturity. The Netherlands



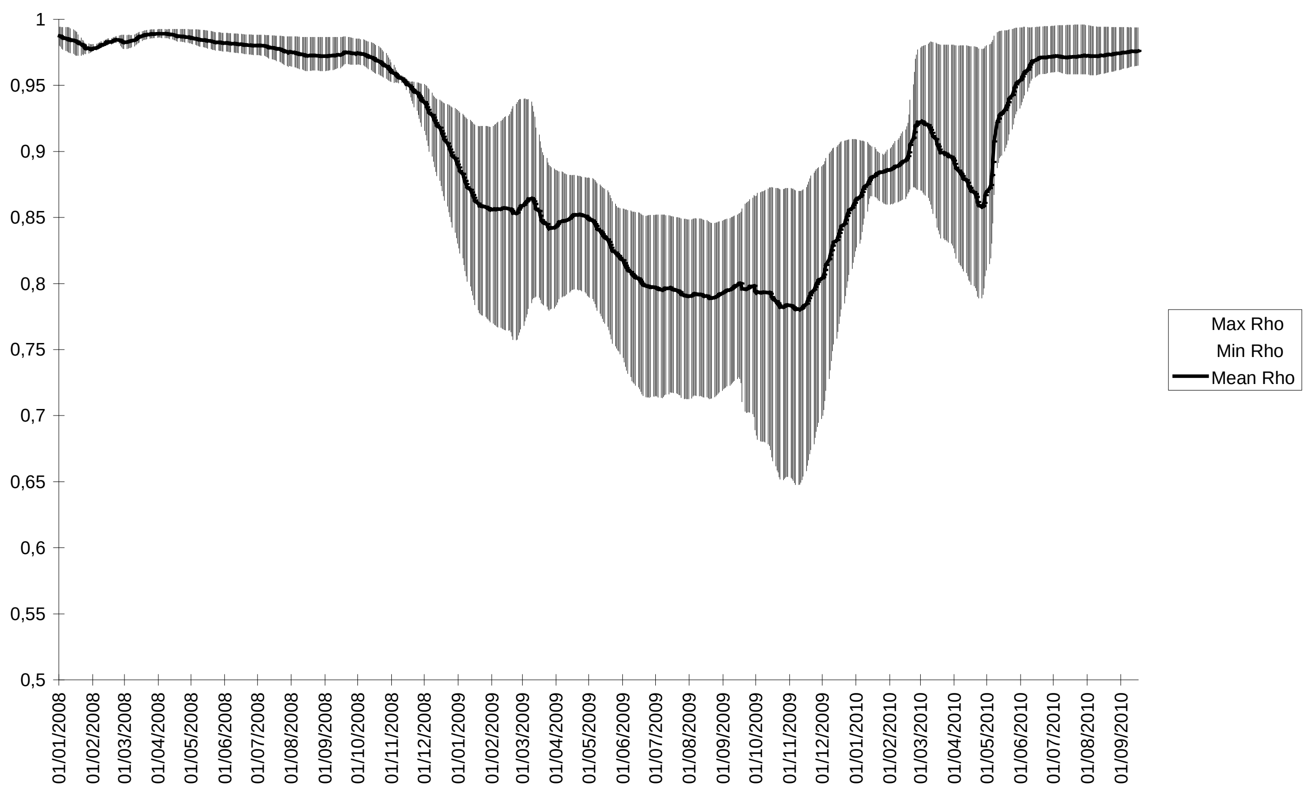
Rank correlation of survival probability. Minimun, maximum and mean values. 12 month moving window. The Netherlands.



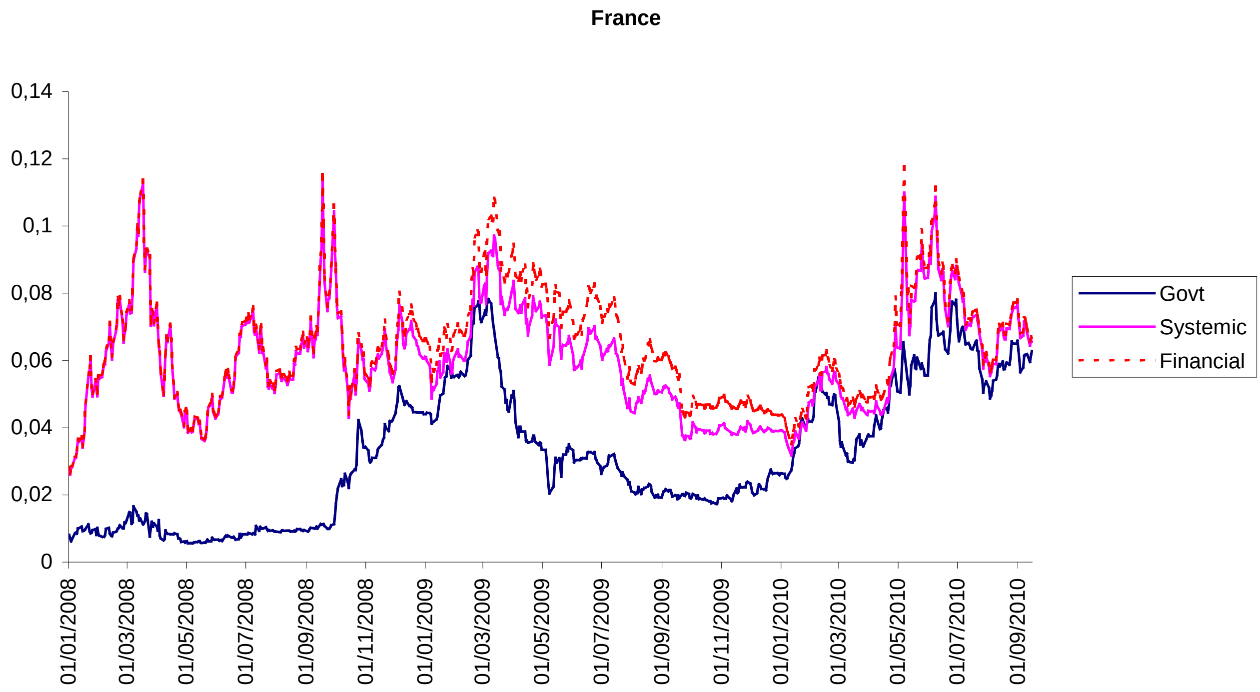
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. The Netherlands



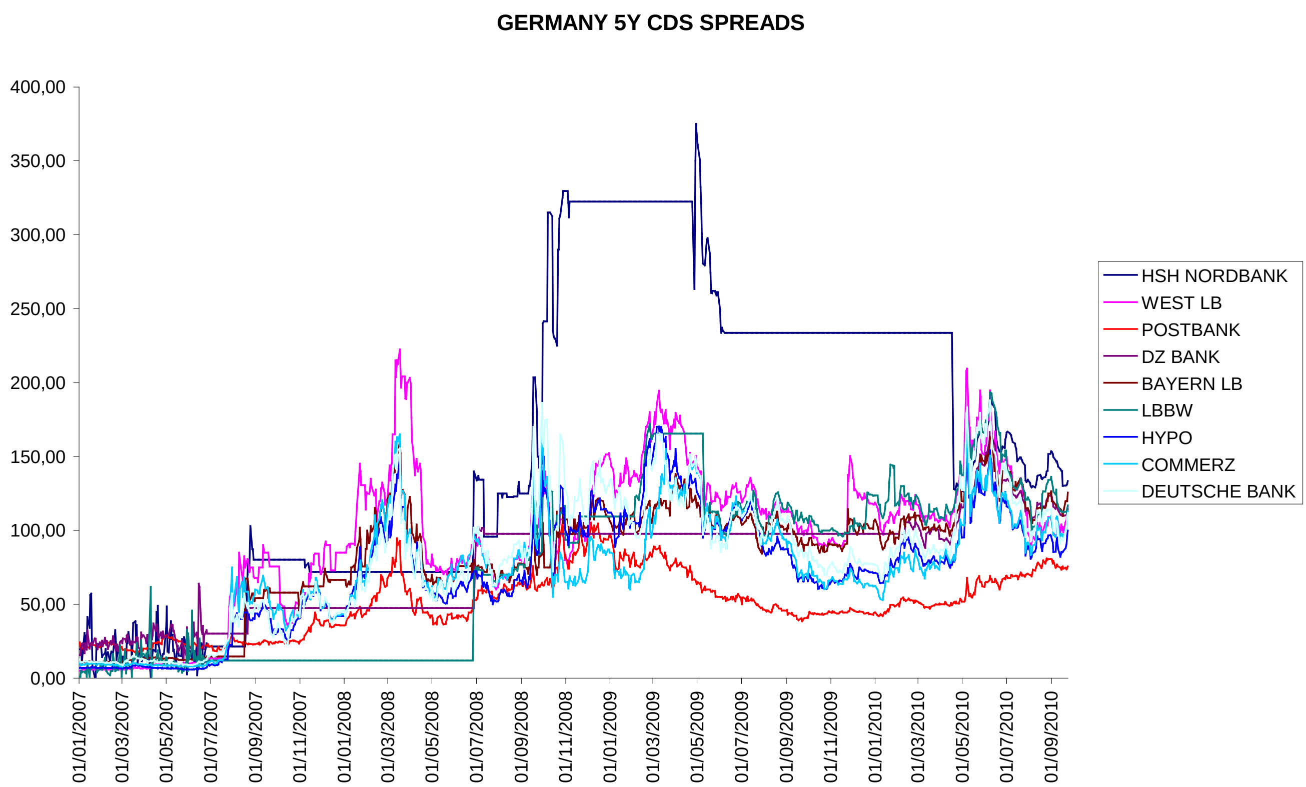
CDS premia. Five year maturity. France



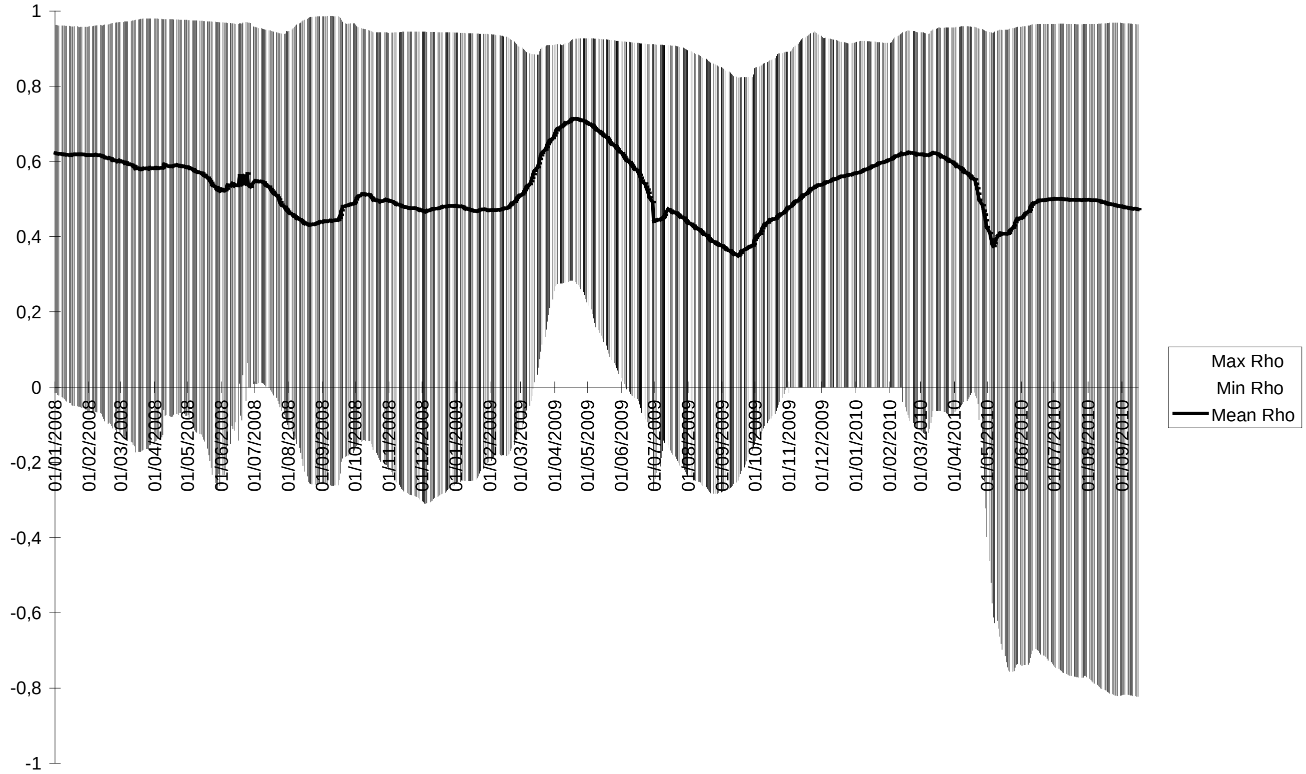
Rank correlation of survival probability: 12 month moving window. France.



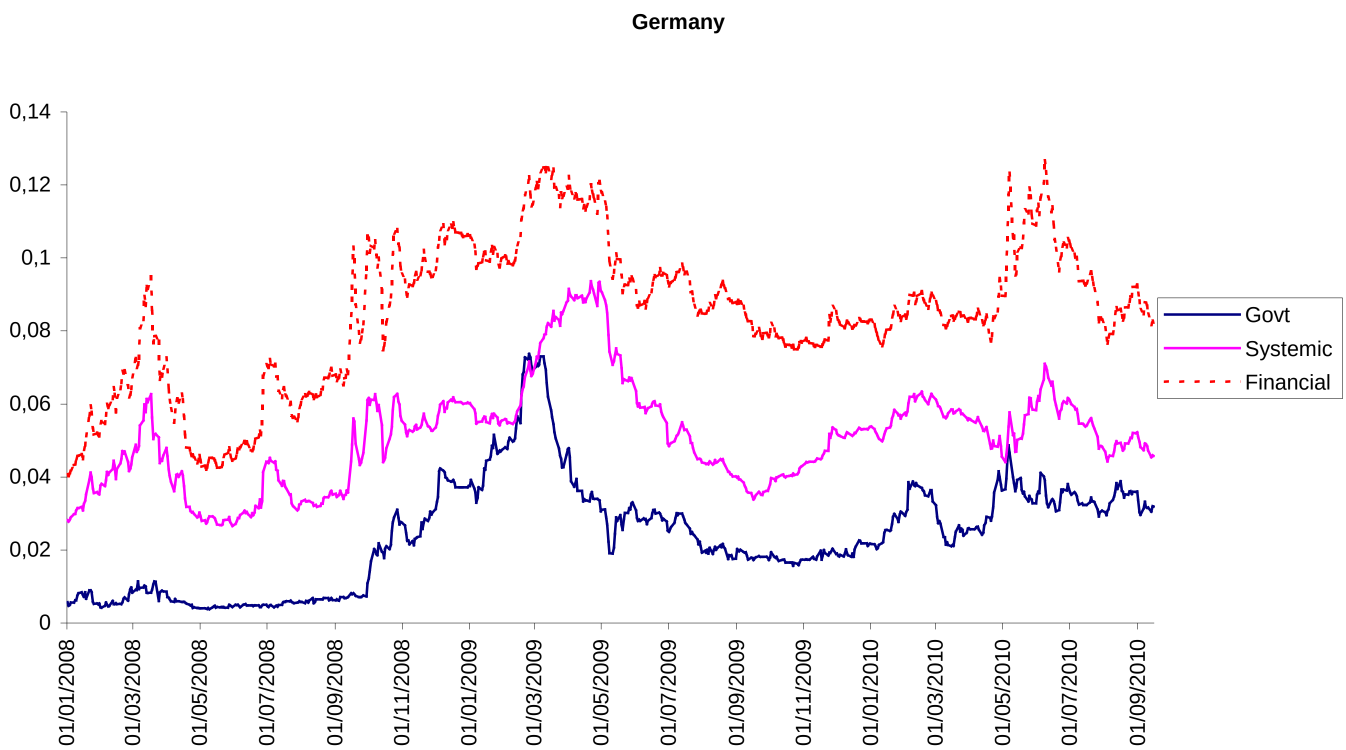
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. France



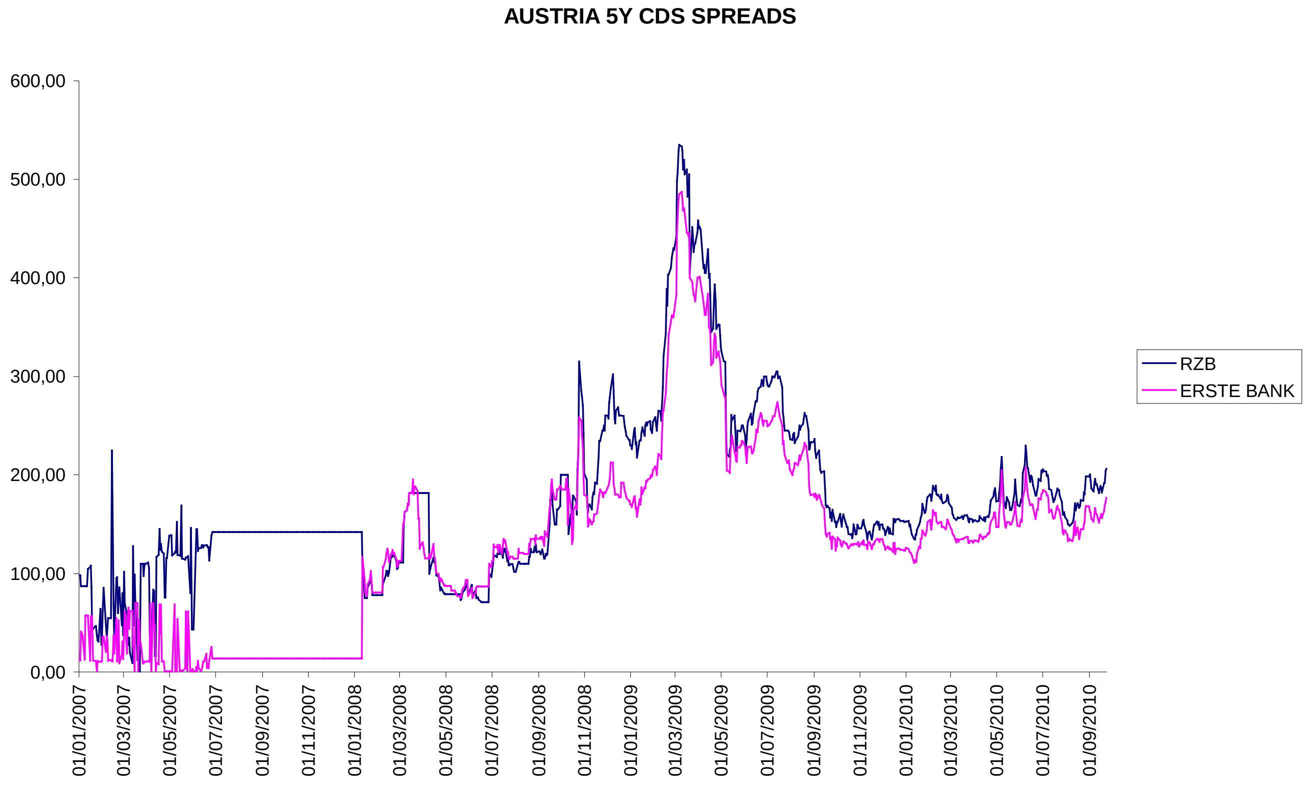
CDS premia. Five year maturity. Germany.



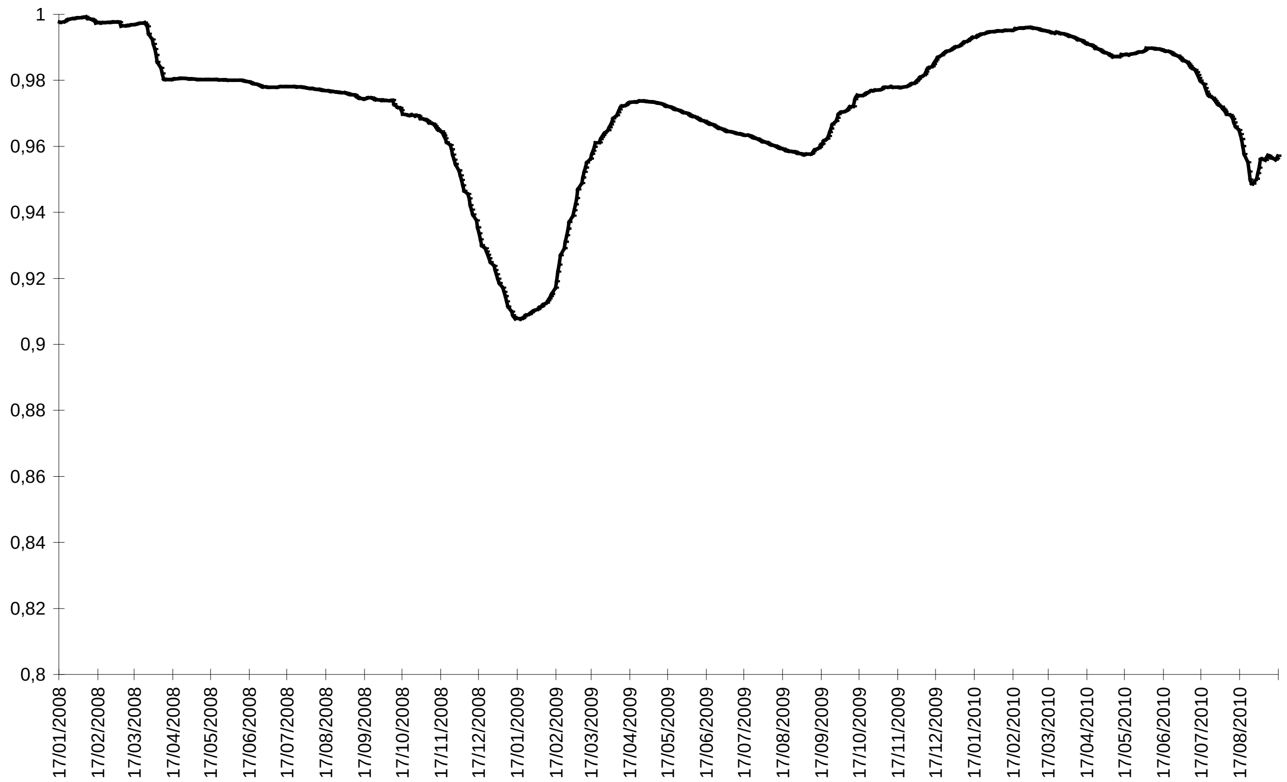
Rank correlation of survival probability. 12 month moving window. Germany.



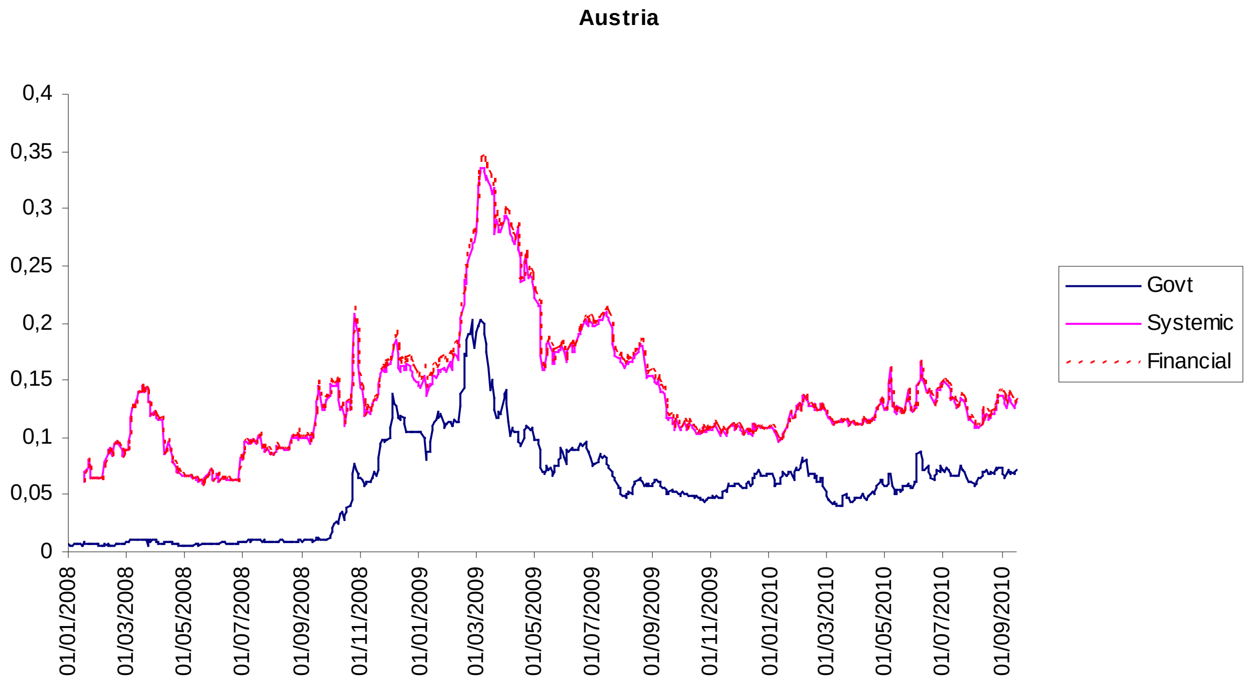
Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Germany



CDS premia. Five year maturity. Austria



Rank correlation of survival probability. 12 month moving window. Austria.



Systemic and average default probability of the financial sector, and Government default probability, over a five year horizon. Austria